Critical phenomena in gravitational collapse

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Summary

Summary

1. Historical development
   - Christodoulou
   - Choptuik
   - Others
2. Key ideas
   - Self-similarity
   - Phase space picture
   - Scaling
3. Recent developments
   - Beyond spherical symmetry
   - Global structure
   - Chaos
4. Conclusions and open questions

Christodoulou

- Nonlinear stability of Minkowski in vacuum
- Cosmic censorship: is it possible to form a naked singularity, visible to distant observers, starting from smooth initial conditions in a self-gravitating system which is regular without gravity?
- Address both problems in a simpler setting: spherical symmetry.
  - Birkhoff theorem in 3+1: no gravitational freedom
  - Add massless real scalar field $\phi(t, r)$
- Results (CMP'86):
  - Small finite data $\Rightarrow$ Minkowski is stable.
  - Large data $\Rightarrow$ Schwarzschild end state.
- What happens in between?
  - Curvature at BH surface $\sim M^{-2}$. Naked singularities?
  - May need self-similarity at the origin, e.g. spherical scalar field

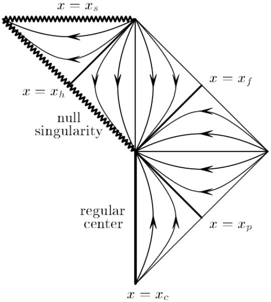
$$\phi(t, r) = f(r/|t|) + \kappa \ln |t|$$

Piran et al.

- Ori & Piran PRL'87, PRD'90
  - Spherically symmetric perfect fluid, $p = k\rho$, $k \ll 1$, self-similarity ansatz

$$\rho(r,t) = |t|^{-2} f(r/|t|)$$

  - Regular at centre and at sonic point $\Leftrightarrow$ regular initial data at $t < 0$.
  - Continue through Cauchy horizon (not unique)



- Goldwirth & Piran, PRD'87: *We present a numerical study of the gravitational collapse of a massless scalar field. We calculate the future evolution of new initial data, suggested by Christodoulou, and we show that in spite of the original expectations these data lead only to singularities engulfed by an event horizon.*

Choptuik

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- 1982–1986 (PhD): scalar field spherical collapse code. Cauchy, fully constrained.
- 1987–1991: Improve accuracy and convergence: adaptive mesh refinement and Richardson extrapolation.

- Choptuik, Goldwirth, Piran CQG'92: compare codes
  ... although the levels of error in the CA and CH results at a given resolution were quite comparable at early retarded times (...), the CA values were significantly more accurate than the CH data once the pulse of scalar field had reached $r = 0$.
- PRL'93: critical phenomena!

Setup

- Fully constrained evolution in Schwarzschild-like coordinates

$$ds^2 = -\alpha^2(t,r)dt^2 + a^2(t,r)dr^2 + r^2 d\Omega^2$$

$$\Phi \equiv \phi', \qquad \Pi \equiv a\dot{\phi}/\alpha$$

$$\dot{\Phi} = \left(\frac{\alpha}{a}\Pi\right)', \qquad \dot{\Pi} = \frac{1}{r^2}\left(r^2\frac{\alpha}{a}\Phi\right)'$$

$$\frac{\alpha'}{\alpha} = \frac{a'}{a} + \frac{a^2-1}{r} = 2\pi r(\Pi^2 + \Phi^2)$$

- One-parameter (p) families of initial conditions:
  - Small p leads to no BH formation (small finite data)
  - Large p produces a BH (large data)

Results

- Bisection in p to $\sim 10^{-15}$ to BH formation threshold:
  - Well-defined p_*: the threshold is not fractal.
  - It is possible to form arbitrarily small black holes.
  - *Scaling*: $M_{BH}(p) \propto (p - p_*)^\gamma$ for $p \gtrsim p_*$.
  - Oscillations in the central region, accumulating at $(r = 0, t = 0)$.
  - *Discrete self-similarity*: $\phi(t, r) \approx \phi(t/e^\Delta, r/e^\Delta)$
  - *Universality*: $\gamma \approx 0.37$, $\Delta \approx 3.44$, same profile $\phi_*(t, r)$ for all families of initial data.
- Conjecture: ϕ_* exact solution with high symmetry and an attractor.
- Comment: Self-similarity is dynamically found, but in a new (discrete) form.

Is this generic?

Independent confirmations:

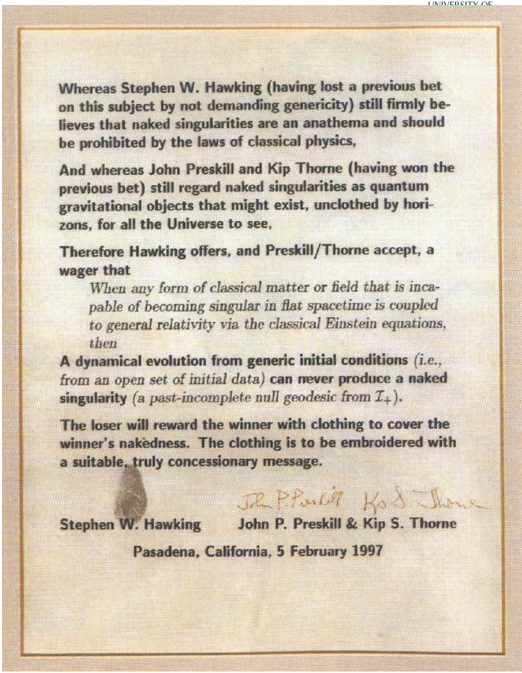
- Gundlach (PRL'95): ϕ_* as solution of eigenvalue problem.
- Hamadé & Stewart (CQG'96): higher precision collapse. Naked!

Phenomenology confirmed in more than 20 other systems:

- Abrahams & Evans (PRL'93): axisymmetric vacuum (DSS)
- Evans & Coleman (PRL'94): perfect fluid, $P = \rho/3$, later $P = k\rho$ (CSS)
- Choptuik, Chmaj & Bizoń (PRL'96): SU(2) Yang-Mills (DSS)
- Liebling & Choptuik (PRL'96): scalar field in Brans-Dicke (CSS/DSS)
- Proca, Dirac, sigma models, ..., Vlasov(?)
- With/without mass, charge, conformal couplings, ...
- Spherical symmetry in 2+1, $n+1$ dimensions

Cosmic censorship?

- Christodoulou (AM'94): Naked singularities in scalar field collapse.
- Christodoulou (AM'99): They are unstable!
- Cosmic censorship is modified: no *stable* naked singularities.
- 1997: Hawking concedes defeat in his famous bet.
- New bet!



Whereas Stephen W. Hawking (having lost a previous bet on this subject by not demanding genericity) still firmly believes that naked singularities are an anathema and should be prohibited by the laws of classical physics,

And whereas John Preskill and Kip Thorne (having won the previous bet) still regard naked singularities as quantum gravitational objects that might exist, unclothed by horizons, for all the Universe to see,

Therefore Hawking offers, and Preskill/Thorne accept, a wager that

> *When any form of classical matter or field that is incapable of becoming singular in flat spacetime is coupled to general relativity via the classical Einstein equations, then*

A dynamical evolution from generic initial conditions *(i.e., from an open set of initial data)* can never produce a naked singularity *(a past-incomplete null geodesic from $\mathcal{I}_+$)*.

The loser will reward the winner with clothing to cover the winner's nakedness. The clothing is to be embroidered with a suitable, truly concessionary message.

Stephen W. Hawking — John P. Preskill & Kip S. Thorne

Pasadena, California, 5 February 1997

Summary

Continuous and discrete self-similarity

Conformal Killing vector ξ

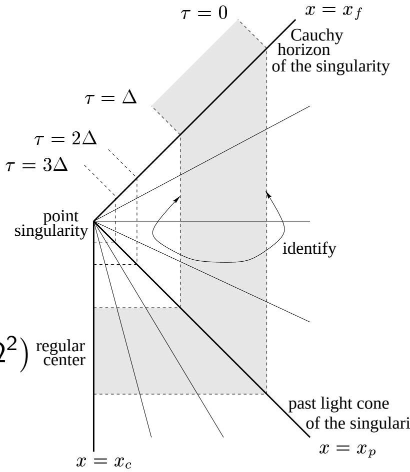
$$\mathcal{L}_\xi \, g_{ab} = -2 \, g_{ab}$$

In spherical symmetry, define adapted coordinates, e.g. starting from Schwarzschild-like coordinates (t, r),

$$x \equiv \frac{r}{-t}, \qquad \tau \equiv -\ln \frac{-t}{T}$$

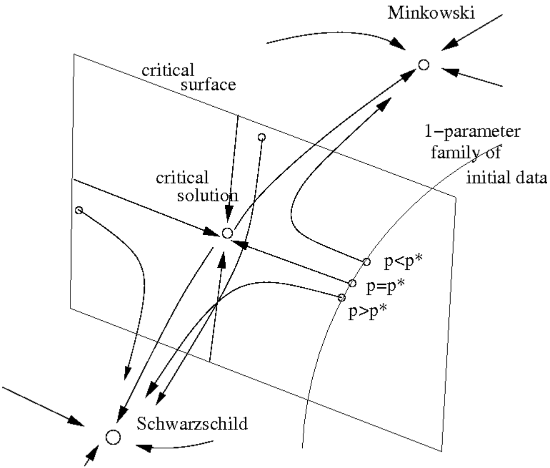
$$ds^2 = e^{-2\tau} \left(A d\tau^2 + 2B d\tau dx + C dx^2 + F d\Omega^2 \right)$$

with $\xi = \partial_\tau$.
CSS: A, B, C, F functions of x only.
DSS: also periodic in τ, period Δ.



Phase space picture

E.g. $\{\Pi(r), \Psi(r)\}$ for the spherical scalar field.



Explains universality and scaling. Note critical solution has one unstable perturbation mode.

Mass scaling

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Near-critical initial data, near the critical solution:

$$\begin{aligned}
\phi(x,\tau) &\simeq \phi_*(x) + \sum_{i=0}^{\infty} C_i(p)\; e^{\lambda_i \tau}\; \phi_i(x) \\
&\simeq \phi_*(x) + \frac{dC_0}{dp}(p_*)\;(p-p_*)\; e^{\lambda_0 \tau}\; \phi_0(x) \\
&\simeq \phi_*(x) + (\text{some constant})\; \phi_0(x) \text{ when AH forms}
\end{aligned}$$

This happens at some τ defined by

$$(p-p_*)\; e^{\lambda_0 \tau_{\mathrm{AH}}} \simeq (\text{some constant})$$

Because $ds^2 = e^{-2\tau} g_{\mu\nu}(x)$,

$$M(p) \sim e^{-\tau_{\mathrm{AH}}(p)} \sim |p-p_*|^{\lambda_0}$$

A standard model?

Numerical approaches:

- Nonlinear evolution and fine-tuning of initial data
- Similarity solution plus linear perturbations

Successes:

- Universality
- Mass scaling
- Universality classes

Open questions:

- Function space and distance measure?
- When is the critical surface smooth? (Always?)
- When does a critical solution exist? (Always? Not in Vlasov?)
- When is it stationary, periodic, CSS, DSS? Any other possibilities?
- Why does it have high symmetry (e.g. spherical?)

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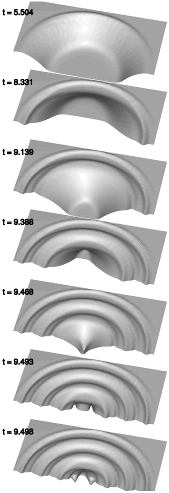
Beyond spherical symmetry: scalar field

Perturbative results:

- Martín-García & Gundlach PRD'99 : All nonspherical perturbations of the Choptuik spacetime decay. Slowest decaying mode is $l = 2$ polar, with $\lambda =$ -0.019(2)+ i 0.55(9).
- Garfinkle, Gundlach & Martín-García PRD'99 : Conjectured scaling law for angular momentum, exponent 0.762(2).
- Gundlach & Martín-García PRD'96 : Conjectured charge scaling, exponent 0.884(1), confirmed by Hod & Piran PRD'96.

Non-linear results:

- Choptuik et al PRD'03, axisymmetry: unstable $l = 2$ polar mode, exponent 0.1–0.4. Critical solution cascade.
- Choptuik et al PRL'04: ansatz $\Phi(t, \rho, z, \varphi) = e^{im\varphi}\Psi(t, \rho, z)$. Isolated m sectors. DSS criticality. $J/M^2 \to 0$ as $M \to 0$.



Beyond spherical symmetry: fluids and vacuum

- Perturbative fluid results: Gundlach PRD'01, perfect fluid with $p = k\rho$, nonspherical perturbations of spherical CSS critical solution
  - $k < 1/9$ (analytical): $l = 1$ axial unstable (centrifugal).
  - $1/9 < k <0.49$: all nonspherical modes stable.
  - $k >0.49$: many unstable polar modes.
  - Note: spherically-stable naked singularity for $k <0.01$ (Harada & Maeda PRD'03, Snajdr CQG'06).
- No nonlinear nonspherical fluid evolutions yet
- Axisymmetric vacuum collapse
  - Abrahams & Evans PRL'93: DSS critical solution
  - Not yet repeated!

Glancing BH collisions

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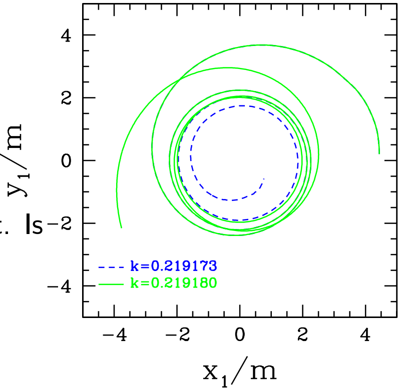
Pretorius & Khurana CQG'07:

- Equal mass BHs. Fine tune boost.
- N circular orbits before merging or dispersing, $N \propto -\gamma \ln |p - p_*|$ with $\gamma \simeq 0.31 - 0.38$
- 1.5% total energy radiated per orbit. Is this an approximate symmetry?
- max N limited by kinetic energy available.



Possible global structure of self-similar spherically symmetric spacetimes

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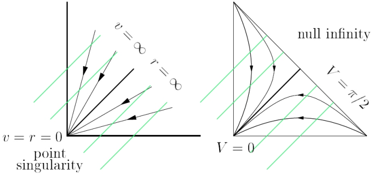
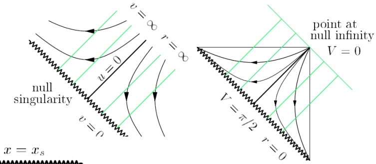
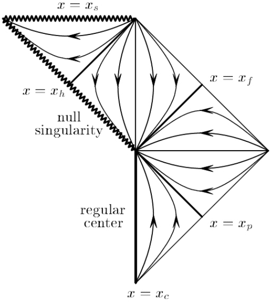
- Recall structure $e^{-2\tau} g_{\mu\nu}(x)$. Kinematical singularity at $\tau = \infty$.
- Self-similarity horizons: null homothetic lines.
- Building blocks: *fan* and *splash* (Gundlach & Martín-García PRD'03)





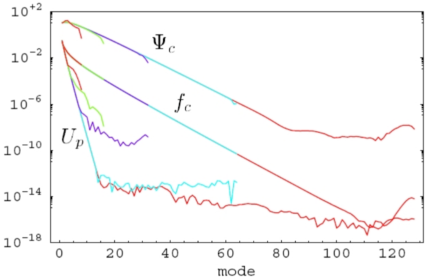
- Example (Ori & Piran PRD'90):

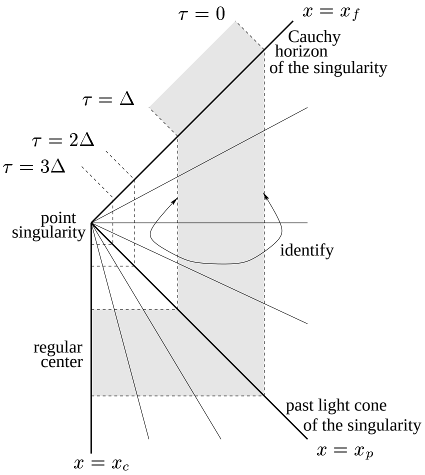


High precision numerical Choptuik spacetime

Martín-García & Gundlach PRD'03

- Three regions
- Psedospectral code. Fourier in τ;
  4^{th} order FD in x.

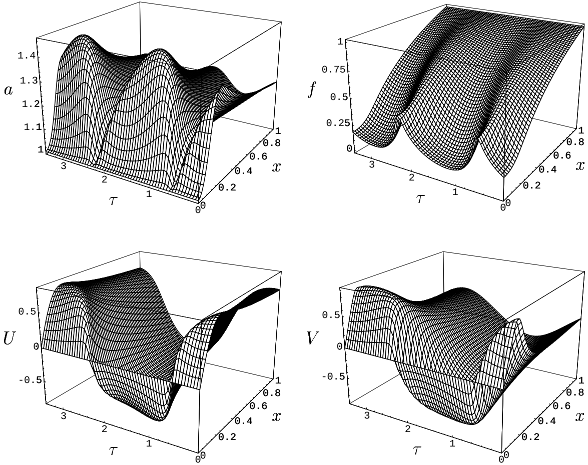




The inner patch

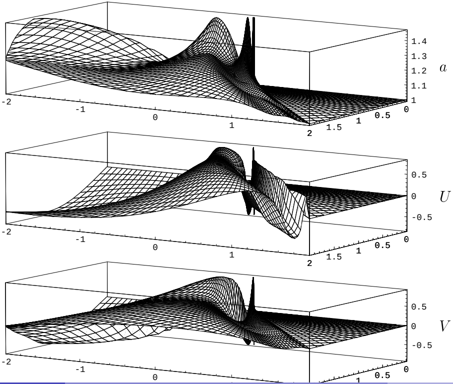
Impose DSS and regularity at centre and past light cone.

$$\Delta = 3.445\,452\,402(3)$$



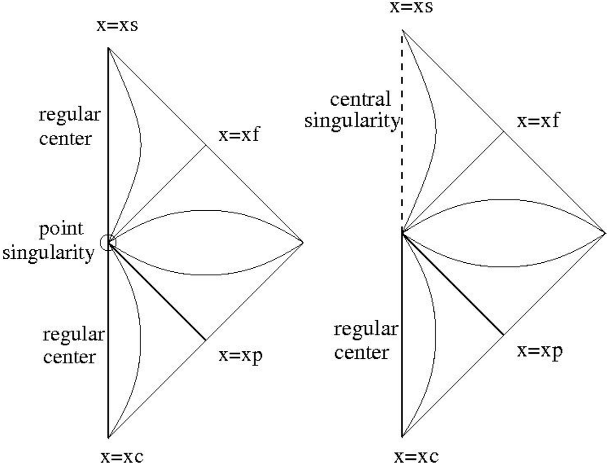
The outer and future patches

- Oscillations pile up at the Cauchy horizon, but decay. Curvature is continuous but non-differentiable. $M/r \sim 10^{-6}$ (to 8 digits)
- Continuation not unique.
- Assuming DSS, one free function (radiation from the point singularity along the CH).
- Unique DSS continuation with regular center (nearly flat):



Global structure of the Choptuik spacetime

All other DSS continuations produce a negative mass singularity at the centre, with no new self-similarity horizon:



4+1 vacuum collapse

Bizoń et al PRD'05, PRL'05, PRL'06

- Gravitational waves in t and r:
- Take

$$ds^2 = -Ae^{-2\delta}dt^2 + A^{-1}dr^2 + \frac{r^2}{4}\left[e^{2B}\sigma_1^2 + e^{2C}\sigma_2^2 + e^{-2(B+C)}\sigma_3^2\right]$$

$$\sigma_1 + i\,\sigma_2 = e^{i\psi}(\cos\theta\, d\phi + i\, d\theta), \qquad \sigma_3 = d\psi - \sin\theta\, d\phi$$

- Triaxial symmetry: 6 copies from permutations of the σ_i
- DSS criticality with $B = C$ (biaxial (3 copies))
- $\Rightarrow$ 3 critical solutions and basins of attraction.
- Boundaries among those are controlled by triaxial DSS codim-2 sols.

Chaos on the critical surface

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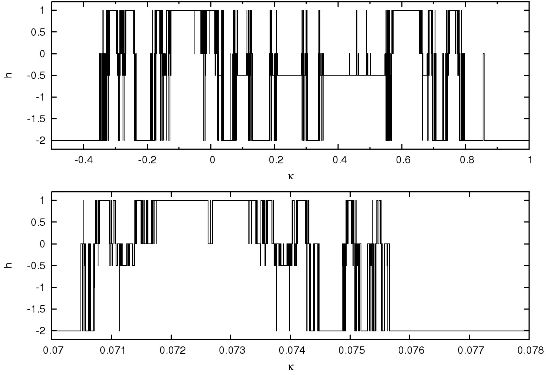
Szybka & Chmaj PRL'08

- Quadruple precision (32 digits) to fine tune two modes.
- Chaotic evolution within the critical surface: which of three DSS end-state? Reported fractal dim 0.68–0.72.
- κ-family of ICs. Possible end-states $h = 1, 1/2, -2$ or 0 (unknown).



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Conclusions and open questions

- It is "easy" to form a naked singularity: fine tune to the BH threshold.
- Process controlled by one self-similar solution.
- Highly structured phase space, possibly chaos.
- Numerical relativity can find new physics.

- Show existence of the Choptuik spacetime.
- How does critical collapse relate to Christodoulou '94,'99?
- What is specifically GR about critical collapse?
- Why DSS?
- What happens beyond spherical symmetry?
- Role of angular momentum? Analogy with magnetic field/temperature?

Gundlach & Martín-García, Living Reviews in Relativity 2007.