Worskhop on Dynamics of Self-Gravitating Matter
Institut Henri Poincaré, Paris

Spin and Topo-spin

Topological aspects of Kerr-Newman spacetimes and their consequences for the Dirac equation

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Some Recent Results in GRQM

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4. M. K. Balasubramanian, “Scalar fields and Spin-1/2 Fields on Mildly Singular Spacetimes," 71 pp., Ph.D. Dissertation, Rutgers –New Brunswick (May 2015).

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- Fundamental objects that are highly singular in both gravity and electromagnetism: point mass and point charge.

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- *“Writers have occasionally noted the possibility that material particles might be considered as singularities of the field. This point of view, however, we cannot accept at all...Every field theory must adhere to the fundamental principle that singularities of the field are to be excluded."*

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- Who were those "writers" who liked singularities?

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- PS: If you dislike a Weyl theory of matter, just wait five minutes!

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- These problems are NOT resolved (in a mathematically rigorous way) by currently known extensions of QM (QED, QFT, ...)

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- Are the predictions of such a theory in agreement with physical experiments?

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- Eigenvalues and Eigenfunctions: Physics becomes Linear Algebra!

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- Schrödinger's equation reproduces Bohr's spectrum!

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- Schrödinger and Dirac shared a Nobel Prize!

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- Negative Continuum: Dirac's Sea, Hole Theory, Positron

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- Gravity is much weaker than electromagnetism, so adding gravitational effects to Dirac's equation should shift spectral lines by a tiny amount, if at all

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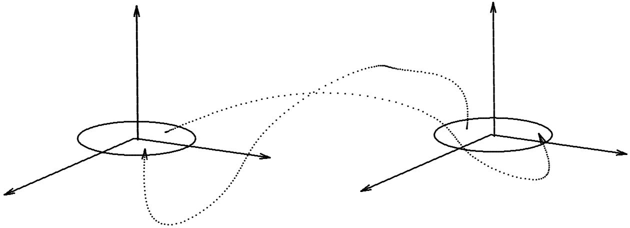

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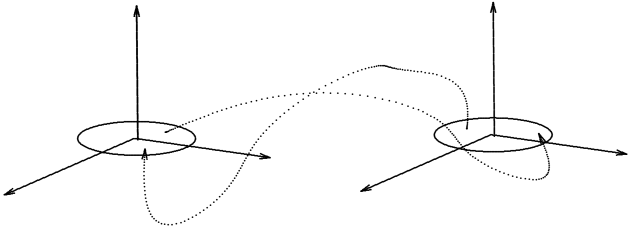

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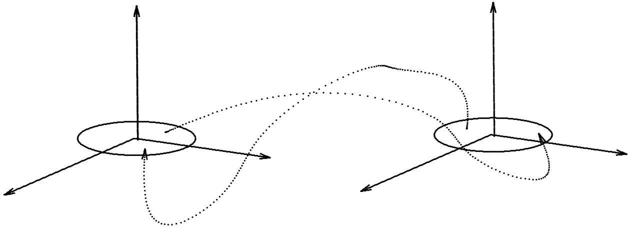

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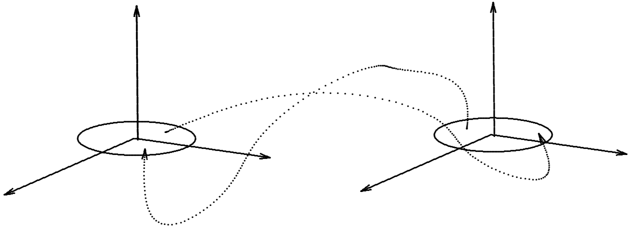

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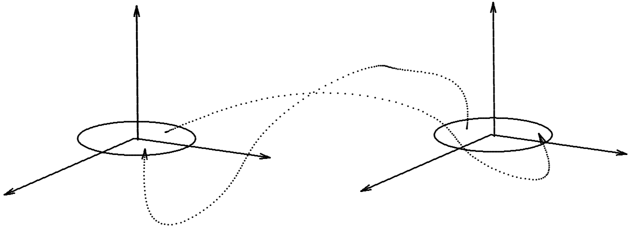

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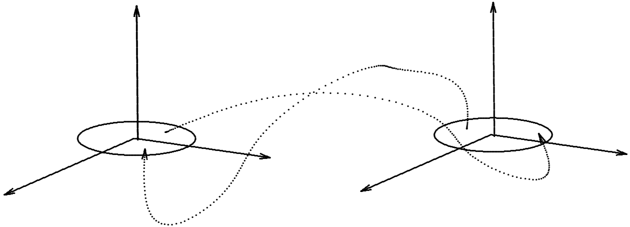

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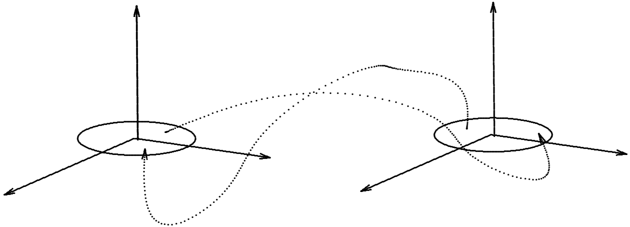

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- zGKN = This spacetime + EM fields on it (P. Appell, 1888)

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- The zero-G limit of the maximally extended Kerr-Newman spacetime yields a flat, but topologically non-trivial, multi-sheeted electromagnetic spacetime.
  This does yield Something New! .... and Interesting:
  Zero-G Kerr-Newman Hydrogen exists!

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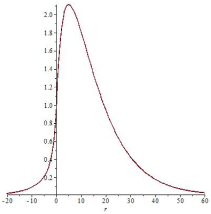

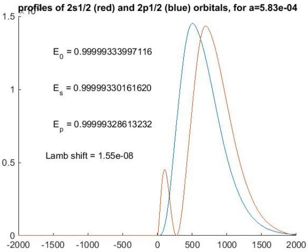

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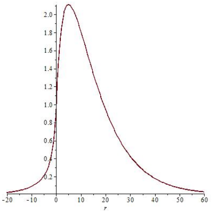

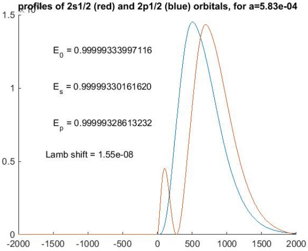

- Excited states. Numerical approximation. Hyperfine splitting and Lamb shift without QED!

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- This resolves the paradox that Dirac's equation "for the electron" also seems to describe "a positron" in many situations, while it is a true one-particle equation.

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- Anti-symmetry of the Dirac Hamiltonian with respect to topo-spin flips gives rise to the *matter-antimatter duality*

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Outlook

- Characterize the discrete Dirac spectrum on zGKN completely
- Do lots of numerical experiments
- Study zero-G general-relativistic Dirac spectrum for "Positronium" (2 rings, 4 sheets?)
- Limit of ∞ many rings may exhibit "ferro-topological phase transition": symmetry breaking (Explanation of broken particle / anti-particle symmetry in our world?).
- Turn gravity back on, within a nonlinear EM theory. (Perturbation method?)

Fin!

THANK YOU FOR LISTENING!

Proof of existence of discrete spectrum

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- $\Psi \in L^2$ iff:
  $\begin{cases} \Omega(-\infty) = -\pi + \cos^{-1}(E), \quad \Omega(\infty) = -\cos^{-1}(E) \\ \Theta(0) = 0, \qquad \Theta(\pi) = -\pi. \end{cases}$

Flows on a finite cylinder

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- Two equilibrium points on each boundary: $f(x_-) = f(x_+) = 0$ and $g_\mu(x_\pm, y) = 0 \implies y \in \{s_\pm, n_\pm\}$

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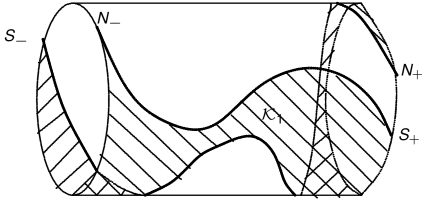

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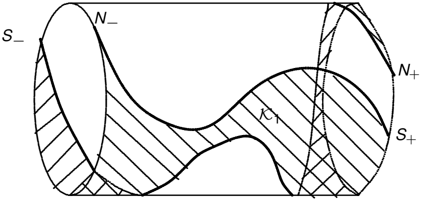

- Saddle-Saddle connector exists iff the corridor collapses.

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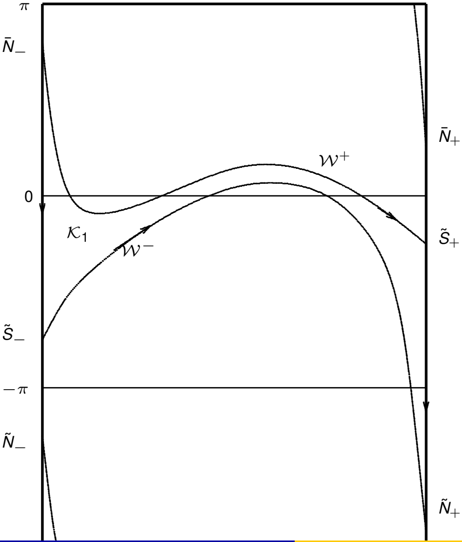
- Construction of barriers to prove existence of corridors with given winding number.

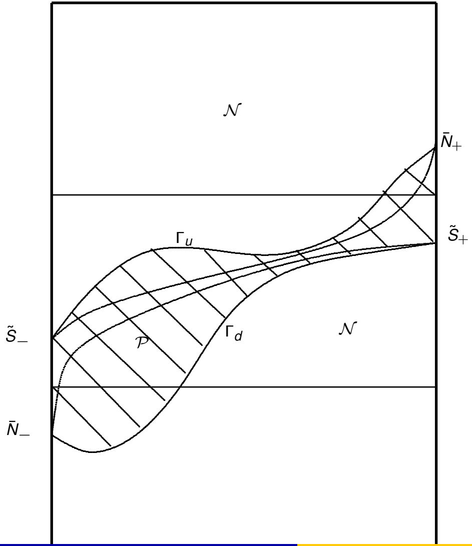
Area and Winding Number for Corridors

Working in the universal cover of the cylinder:

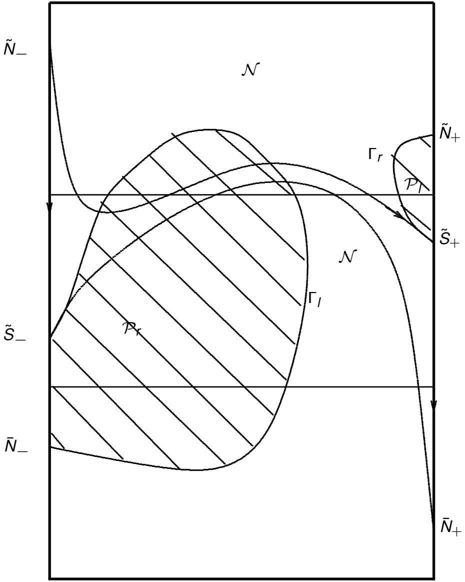


Topology of Nullclines

Orbits must increase while in the shaded region



Change in Nullcline Topology and Corridor Winding



Barrier construction