Boundedness of massless scalar waves on Reissner-Nordström interiors

Anne Franzen

under supervision of
Mihalis Dafermos

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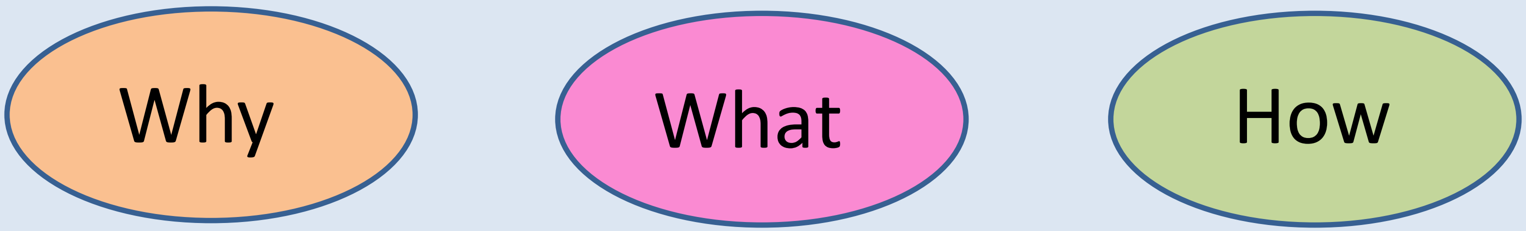
Boundedness, $|\phi| < C$, on fixed interior Reissner-Nordström backgrounds



Sneak Preview:

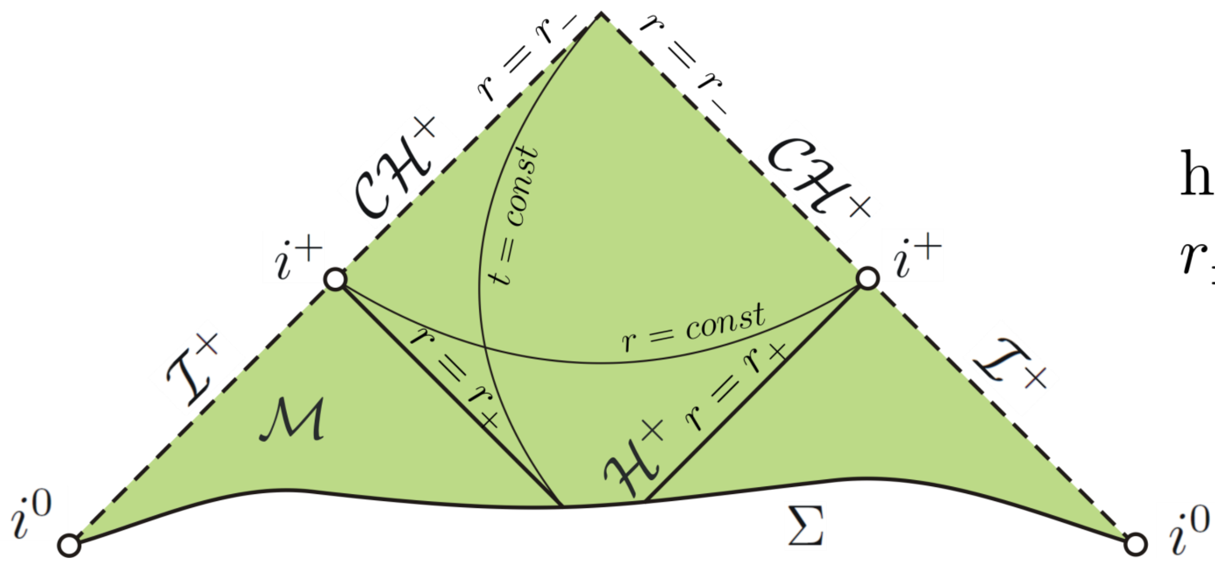
- Validity of the **Strong Cosmic Censorship Conjecture**
- Reissner-Nordström as a proxy for Kerr
- Investigation of $\Box_g \phi = 0$ as a "poor man's" linearisation to the Einstein field equations, using weighted energy estimates and commutation of angular momentum operators

Reissner-Nordström spacetimes

(r, t, φ, θ) coordinates:

$$g = -\left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)^{-1} dr^2 + r^2 d\sigma_{\mathbb{S}}^2,$$

$$d\sigma_{\mathbb{S}}^2 = \sin^2\theta d\varphi^2 + d\theta^2$$



horizons:
$r_\pm = M \pm \sqrt{M^2 - e^2}$

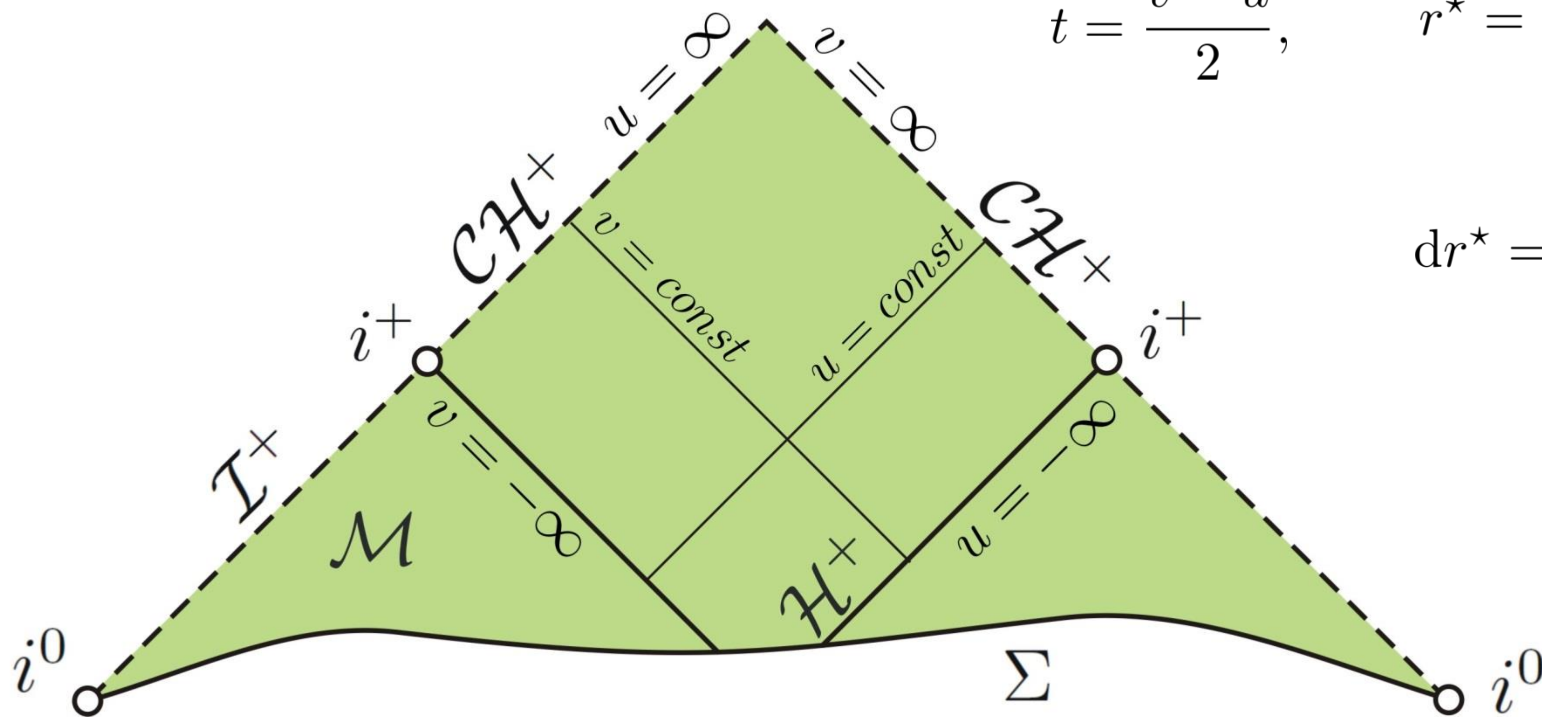
surface gravities:
$\kappa_\pm = \frac{r_\pm - r_\mp}{2r_\pm^2}$

Note that $\frac{\partial}{\partial_t}$ is a Killing vector, spacelike in the interior.

Reissner-Nordström spacetimes

(u, v, φ, θ) double null coordinates:

$$g = -\Omega^2(u,v)\mathrm{d}u\mathrm{d}v + r^2(u,v)\mathrm{d}\sigma_{\mathbb{S}}^2, \qquad \mathrm{d}\sigma_{\mathbb{S}}^2 = \sin^2\theta \mathrm{d}\varphi^2 + \mathrm{d}\theta^2,$$

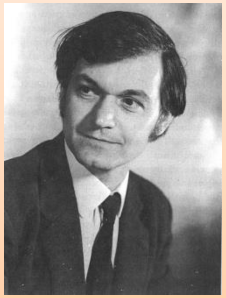


$$t = \frac{v-u}{2}, \qquad r^\star = \frac{v+u}{2},$$

$$\mathrm{d}r^\star = -\frac{\mathrm{d}r}{\Omega^2}.$$

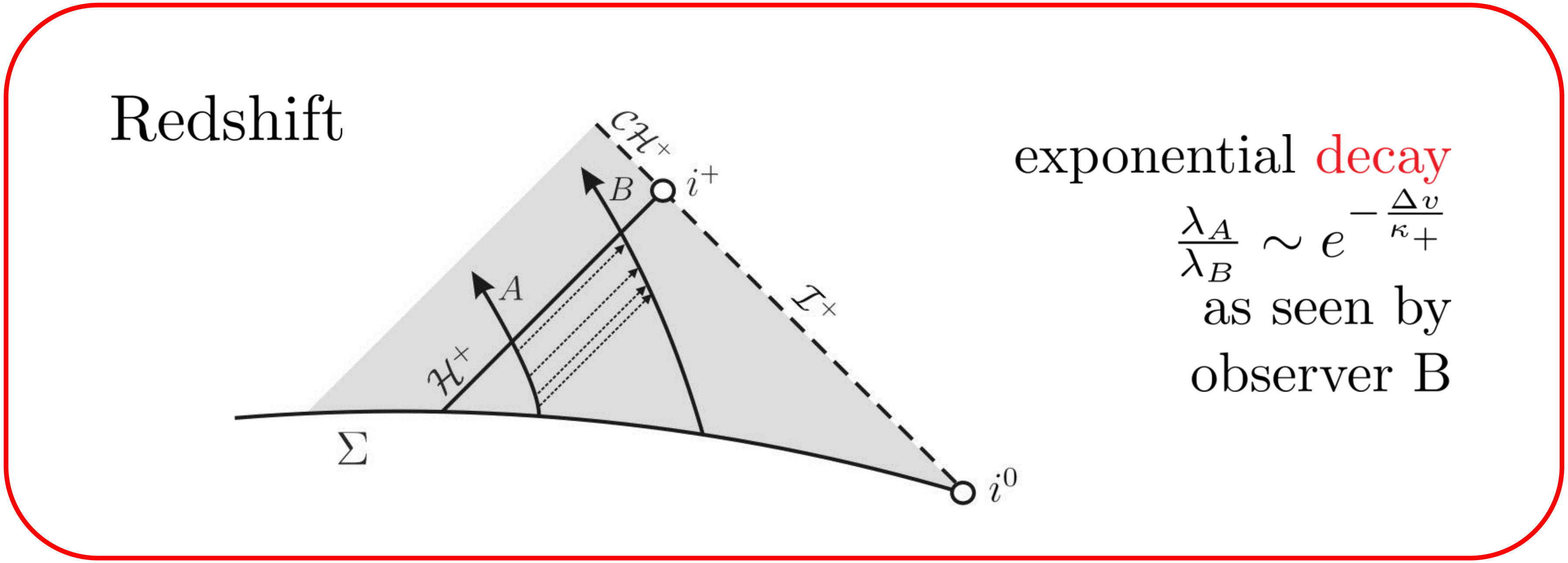
Motivation

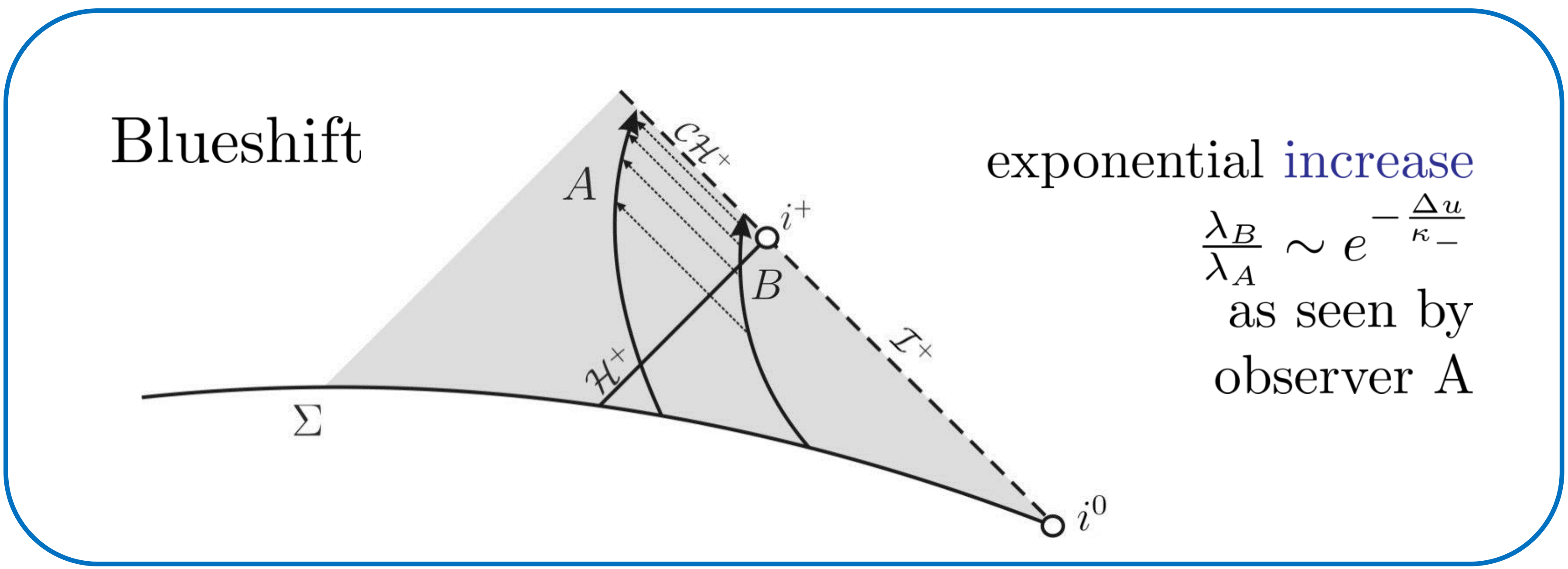
The Strong Cosmic Censorship Conjecture



Roger
Penrose

Infalling radiation is likely to convert into a curvature singularity due to the divergence of energy caused by the infinite blueshift effect.


Redshift
exponential decay
$\frac{\lambda_A}{\lambda_B} \sim e^{-\frac{\Delta v}{\kappa_+}}$
as seen by
observer B
$\mathcal{CH}^+$
i^+
B
A
$\mathcal{H}^+$
$\mathcal{I}^+$
Σ
i^0


Blueshift
exponential increase
$\frac{\lambda_B}{\lambda_A} \sim e^{-\frac{\Delta u}{\kappa_-}}$
as seen by
observer A
$\mathcal{CH}^+$
A
i^+
B
$\mathcal{H}^+$
$\mathcal{I}^+$
Σ
i^0

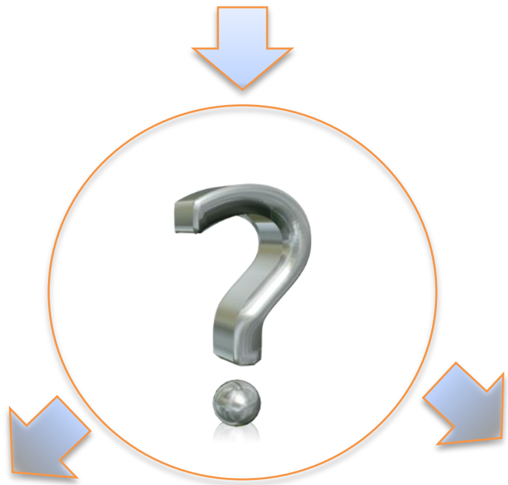
Motivation

The Strong Cosmic Censorship Conjecture

Demetrios Christodoulou

"Generic asymptotically flat initial data for Einstein-Maxwell spacetimes have a maximal future development which is inextendible as a suitably regular Lorentzian manifold."

Reissner-Nordström spacetime
is extendible but not generic.



The Strong Cosmic Censorship Conjecture might not hold.

Extendibility property of Reissner-Nordström spacetime might not be stable.

Instability investigations:

Numerically:

- Penrose & Simpson: perturbation leads to inextendibility as C^0 metric
  $\rightarrow$ *strong* spacetime singularity

Null-fluid models:

- Hiscock: ingoing null dust via RN-Vaidya metric leads to diverging tidal forces, well behaved curvature tensors
  $\rightarrow$ *weak* null singularity

- Poisson & Israel: in and outgoing null fluids, influx gets blueshifted , outflux causes mass function to diverge at $\mathcal{CH}^+$
  $\rightarrow$ *curvature* singularity

- Ori: two patches of Vaidya solutions matched along a thin null layer of dust, metric tensor well behaved
  $\rightarrow$ *weak* singularity

Instability investigations:

Partial Differential Equations:

- McNamara: Reissner-Nordström fixed mode stability at $\mathcal{CH}^+$ and instability for transverse derivative for specific initial data
- Dafermos: spherically symmetric Einstein-Maxwell-scalar field, metric extendible as C^0 but not as C^1 $\rightarrow$ *weak* singularity
- A.F.:

We investigate $\Box_g \phi = 0$ as a proxy for the full non-linear Einstein-Maxwell equations.

Main results

<u>Main Theorem</u>

On subextremal Reissner-Nordström spacetime $(\mathcal{M}, g)$, with mass M and charge e and $M > |e| \neq 0$, let ϕ be a solution of the wave equation $\Box_g \phi = 0$ arising from sufficiently regular Cauchy data on a two-ended asymptotically flat Cauchy surface Σ. Then

$$|\phi| \leq C$$

globally in the black hole interior, in particular up to and including the Cauchy horizon $\mathcal{CH}^+$.

Main results

Energy Theorem

On subextremal Reissner-Nordström spacetime $(\mathcal{M}, g)$, with mass M and charge e and $M > |e| \neq 0$, let ϕ be a solution of the wave equation $\Box_g \phi = 0$ arising from sufficiently regular Cauchy data on a two-ended asymptotically flat Cauchy surface Σ. Then for all values of Eddington-Finkelstein coordinates (u_{fix}, v_{fix}) in the black hole interior

$$\int_{\mathbb{S}^2} \int_{v_{fix}}^{\infty} \left[v^p (\partial_v \phi)^2 (u_{fix}, v) + |\nabla\!\!\!/ \phi|^2 (u_{fix}, v) \right] \mathrm{d}v \mathrm{d}\sigma_{\mathbb{S}}^2 \quad \leq \quad E, \quad for \quad v_{fix} \geq 1,$$

$$\int_{\mathbb{S}^2} \int_{u_{fix}}^{\infty} \left[u^p (\partial_u \phi)^2 (u, v_{fix}) + |\nabla\!\!\!/ \phi|^2 (u, v_{fix}) \right] \mathrm{d}u \mathrm{d}\sigma_{\mathbb{S}}^2 \quad \leq \quad E, \quad for \quad u_{fix} \geq 1.$$

Preliminaries

Energy currents and vector field method

Matter field Lagrangian:

$$\mathcal{L}(\phi, \mathrm{d}\phi, g^{-1}) = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi,$$

From the Euler-Lagrange equations:

$$\Box_g\phi = 0.$$

Stress energy-momentum tensor of massless scalar field:

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi$$

Energy conservation:

$$\nabla^\mu T_{\mu\nu} = (\Box_g\phi)\mathrm{d}\phi = 0.$$

Preliminaries

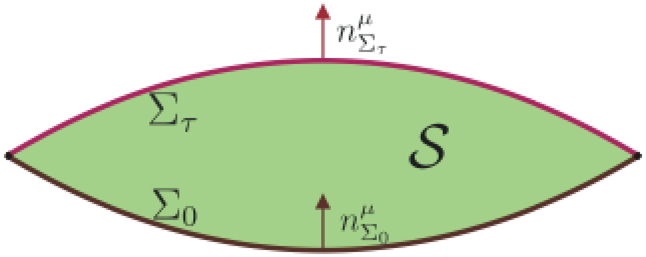
Define the currents:

$$J_\mu^V(\phi) \doteq T_{\mu\nu}(\phi)V^\nu, \quad K^V(\phi) \doteq \nabla^\mu J_\mu(\phi) = \frac{1}{2}(\mathcal{L}_V g)^{\mu\nu} T_{\mu\nu}(\phi).$$

V timelike, n_Σ^μ normal vector, Σ spacelike or null $\Rightarrow J_\mu^V(\phi) n_\Sigma^\mu \geq 0$.

The divergence theorem

To obtain Energy Theorem use versions of the divergence theorem. Consider a spacetime region $\mathcal{S}$ which is bounded by the homologous hypersurfaces Σ_τ and Σ_0 and obtain



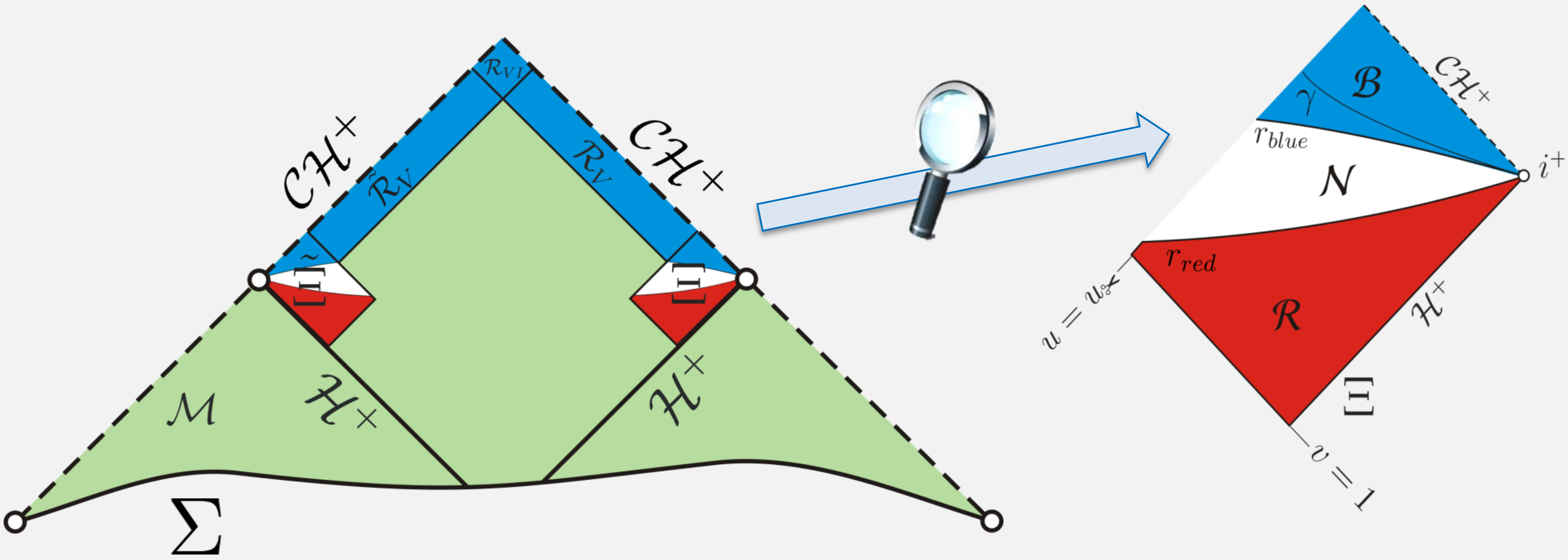
$$\int_{\Sigma_\tau} J_\mu^V(\phi) n^\mu_{\Sigma_\tau} \mathrm{dVol}_{\Sigma_\tau} + \int_{\mathcal{S}} \nabla^\mu J_\mu(\phi) \mathrm{dVol}$$

$$= \int_{\Sigma_0} J_\mu^V(\phi) n^\mu_{\Sigma_0} \mathrm{dVol}\Sigma_0.$$

Sketch of the proof

Separating into different regions

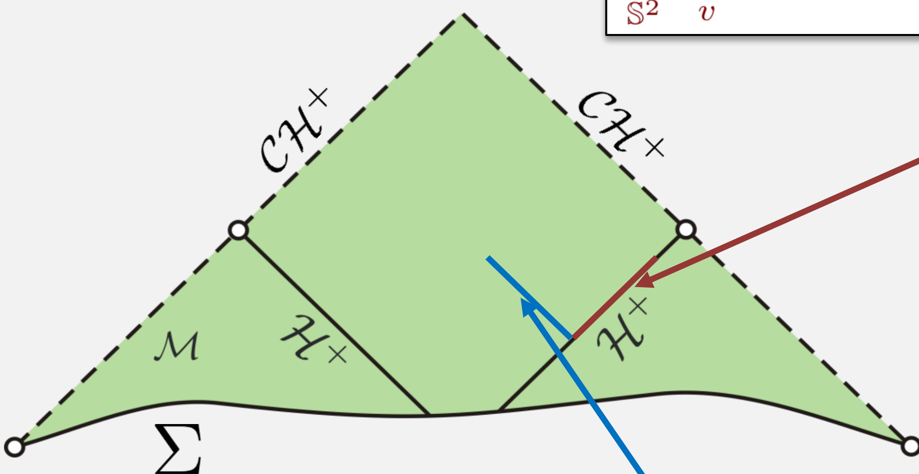
In order to obtain the result of the Energy Theorem we separate our spacetime different regions and use the divergence Theorem in these.



Putting together work of P. Blue and A. Soffer [1] on integrated local energy decay, M. Dafermos and I. Rodnianski on the redshift [2] and V. Schlue [3] on improved decay in exterior black hole regions, results into

decay along the event horizon,

$$\int_{\mathbb{S}^2} \int_v^{v+1} \left[(\partial_v \phi)^2(-\infty, v) + |\nabla\!\!\!/ \phi|^2(-\infty, v) \right] \mathrm{d}v \mathrm{d}\sigma_{\mathbb{S}}^2 \quad \leq \quad C_0 v^{-2-2\delta},$$



with angular derivatives

$$|\nabla\!\!\!/ \phi|^2 = \frac{1}{r^2} \left[(\partial_\theta \phi)^2 + \frac{1}{\sin^2\theta} (\partial_\varphi \phi)^2 \right],$$

and boundedness along a null segment transverse to the event horizon,

$$\int_{\mathbb{S}^2} \int_{-\infty}^{u_\diamond} \left[(\partial_u \phi)^2(u, v_{fix}) + |\nabla\!\!\!/ \phi|^2(u, v_{fix}) \right] \mathrm{d}u \mathrm{d}\sigma_{\mathbb{S}}^2 \quad \leq \quad D_0(u_\diamond, v_{fix}),$$

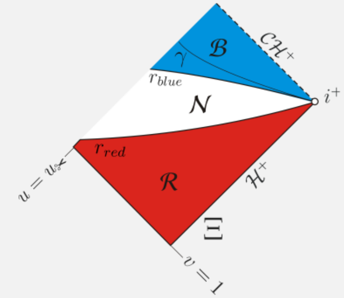
$$\sup_{-\infty \leq u \leq u_\diamond} \int_{\mathbb{S}^2} (\phi)^2(u, v_{fix}) \mathrm{d}\sigma_{\mathbb{S}}^2 \quad \leq \quad D_0(u_\diamond, v_{fix}).$$

References

[1] Blue, P. and Soffer, A. (2007) *Phase Space Analysis on some Black Hole Manifolds. arXiv:0511281 [math.AP]*

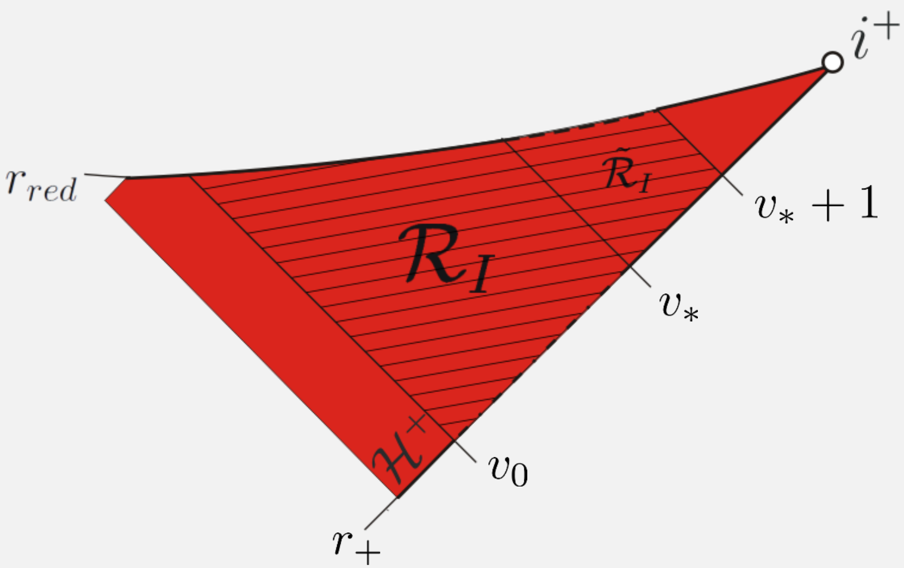
[2] Dafermos, M. and Rodnianski, I. (2005). A proof of Price's law for the collapse of a self-gravitating scalar field. *I*nvent. Math. **162**, 381-457.

[3] Schlue, V. (2010). Decay of linear waves on higher-dimensional Schwarzschild black holes. *Analysis & PDE,* **6**, **3**, *515-600.arXiv:gr-qc/1012.5963*



Redshift region $\mathcal{R} = \{r_{red} \leq r \leq r_+\}$

We make use of the fact, that the surface gravitiy κ_+ of $\mathcal{H}^+$ is positive.



Region $\mathcal{R}$ is characterized by the fact that for r_{red} close enough to $\mathcal{H}^+$ there exists a vector field N such that

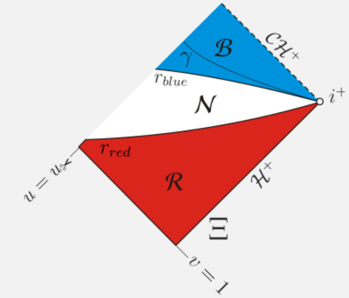
$$bJ^N_\mu(\phi)N^\mu \leq K^N(\phi).$$

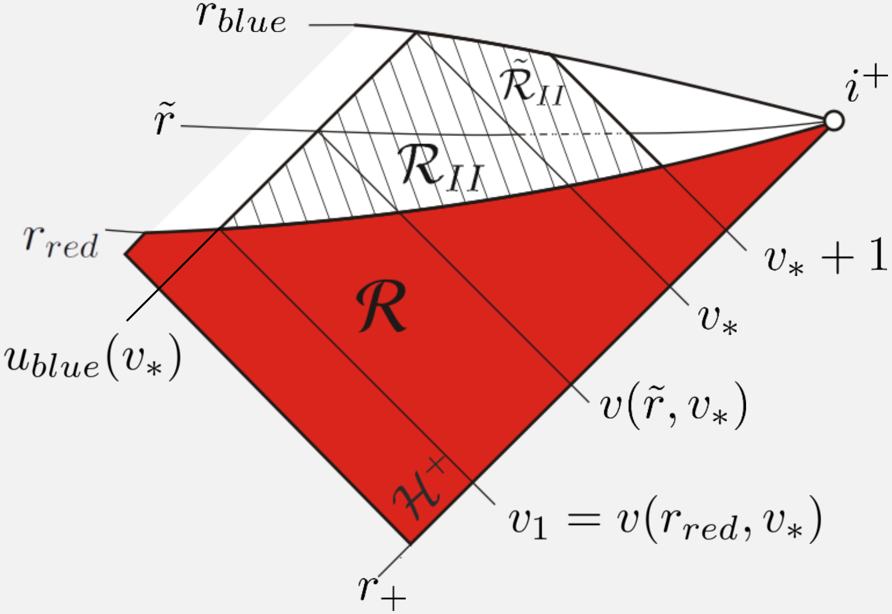
(Theorem by Dafermos and Rodnianski,)

Dafermos, M. and Rodninanski, I. (2013). Lectures on black holes and linear waves. *Clay Mathematics Proceedings, Amer. Math. Soc.* **17**, *97-205. arxiv:gr-qc/0811.0354.*

$$\Rightarrow \int_{\mathbb{S}^2} \int_{v_*}^{v_*+1} J^N_\mu n^\mu_r \mathrm{dVol}_r \mathrm{d}\sigma^2_{\mathbb{S}} \quad \leq \quad C v_*^{-2-2\delta}, \quad \text{matching the decay on } \mathcal{H}^+$$

Noshift region $\mathcal{N} = \{r_{blue} \le r \le r_{r_red}\}$





For the vector field

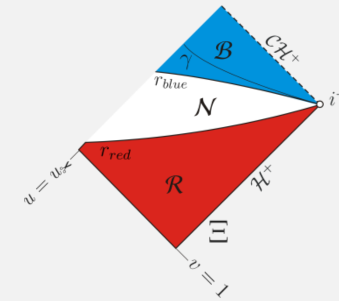
$$-\partial_r = \frac{1}{\sqrt{\Omega^2}}(\partial_u + \partial_v)$$

we can control the bulk

$$|K^{-\partial_r}(\phi)| \le B J_\mu^{-\partial_r}(\phi) n_r^\mu.$$

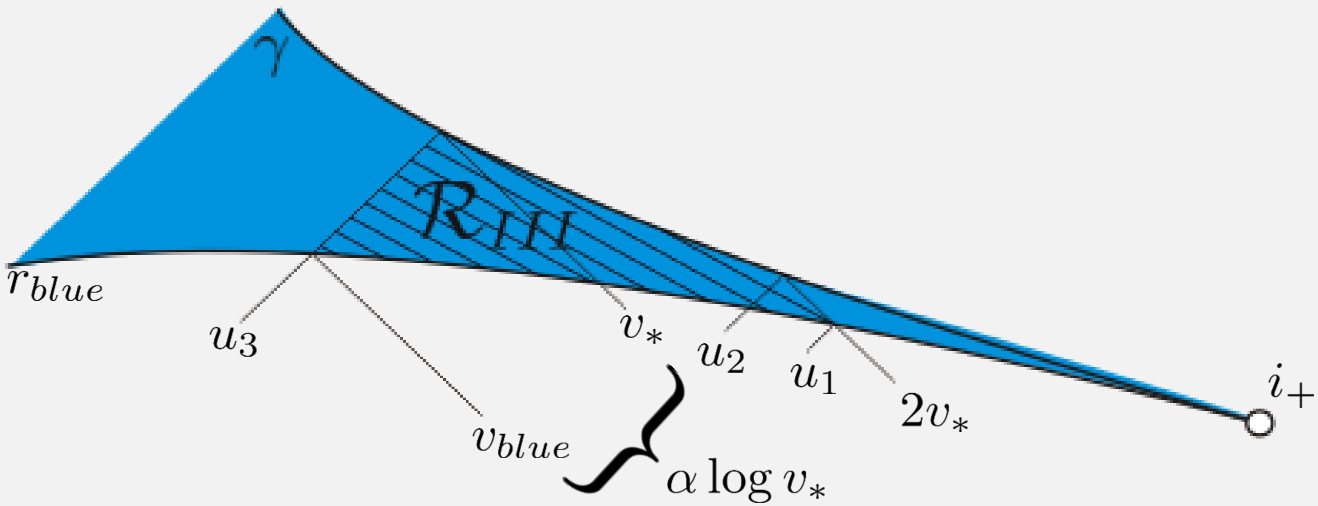
- Timelike currents contain all derivatives.
- Uniformity of B is given since $K^{-\partial_r}$ is invariant under translations along ∂_t.

$$\Rightarrow \int_{\mathbb{S}^2} \int_{v_*}^{v_*+1} J_\mu^{-\partial_r} n_r^\mu \mathrm{dVol}_r \mathrm{d}\sigma_{\mathbb{S}}^2 \quad \le \quad C v_*^{-2-2\delta}, \quad \text{matching the decay on } \mathcal{H}^+$$



Blueshift region $\quad \mathcal{B} = \{r_- \leq r \leq r_{blue}\}$,

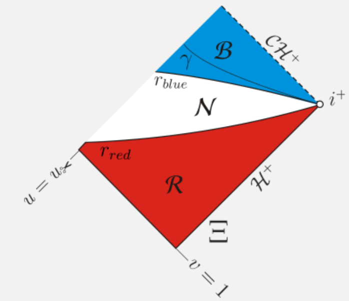
where is $\mathcal{B}$ separated by a suitable hypersurface γ.



$J^-(\gamma) \cap \mathcal{B}$ We use a vector field

$$S_0 = r^q \partial_{r^\star},$$

for which the bulk K^{S_0} is positive for big enough q and for r_{blue} close enough to $\mathcal{CH}^+$.



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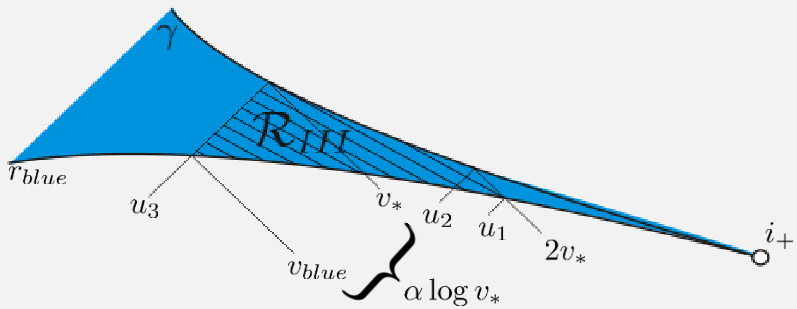
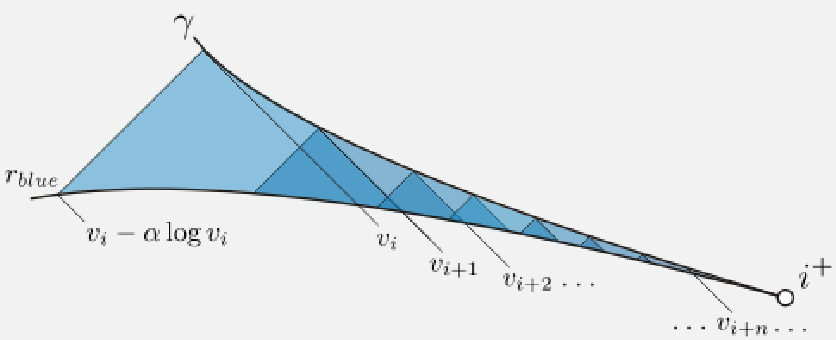


$J^-(\gamma) \cap \mathcal{B}$ Estimate for dyadic length at the expense of one polynomial power

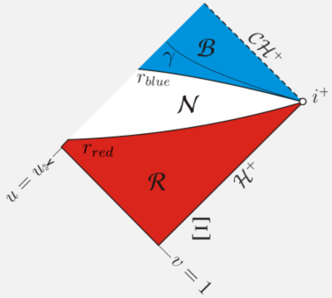
$$\Rightarrow \int_{\mathbb{S}^2} \int_{v_*}^{2v_*} J_\mu^{S_0} n_\gamma^\mu \mathrm{dVol}_\gamma \mathrm{d}\sigma_{\mathbb{S}}^2 \quad \leq \quad C v_*^{-1-2\delta}.$$

Introducing a dyadic sequence $v_i \in [v_*, \infty)$, with $i \in \mathbb{N}_0$, such that $v_{i+1} = 2v_i$, by summing and then weighting the above with v^p we obtain the weighted energy estimate

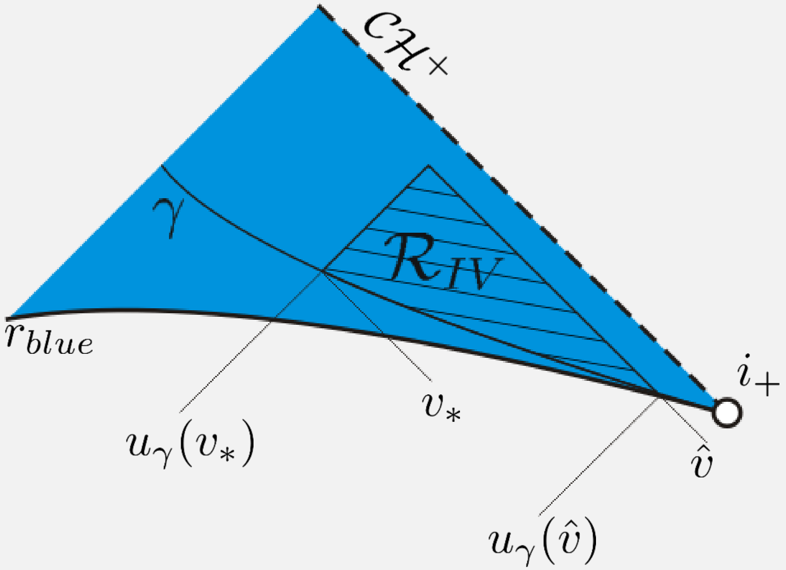


$$\Rightarrow \int_{\mathbb{S}^2} \int_{v_*}^{\infty} v^p J_\mu^{S_0} n_\gamma^\mu \mathrm{dVol}_\gamma \mathrm{d}\sigma_{\mathbb{S}}^2 \quad \leq \quad C v_*^{-1-2\delta+p}.$$

Blueshift region $\mathcal{B} = \{r_- \le r \le r_{blue}\}$,



where is $\mathcal{B}$ separated by a suitable hypersurface γ.



$J^+(\gamma) \cap \mathcal{B}$

pointwise decay estimates on Ω^2

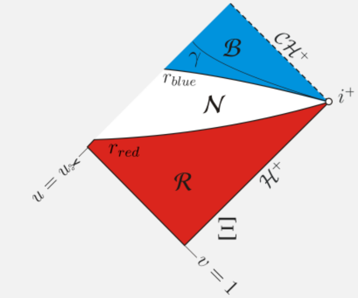
$$\Omega^2(\bar{u}, \bar{v}) \le C|u_\gamma(\bar{v})|^{-\beta\alpha} e^{-\beta|u_\gamma - \bar{u}|},$$

for $(\bar{u}, \bar{v}) \in J^+(\gamma)$

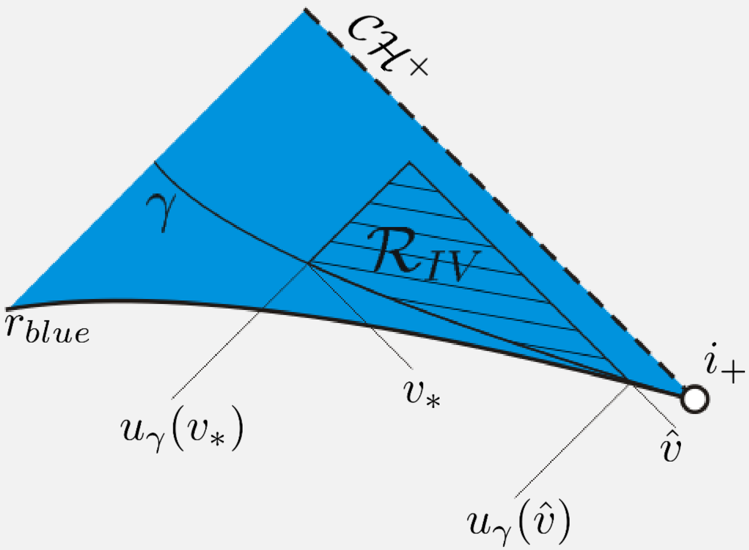
$$0 < \beta \le -\frac{\partial_{u,v}\Omega}{\Omega}, \quad \alpha > 1, \quad \alpha\beta > p + 1$$

By the choice of γ the spacetime volume in $J^+(\gamma)$ is finite, $\mathrm{Vol}(J^+(\gamma)) < C$.

Blueshift region $\quad \mathcal{B} = \{r_- \leq r \leq r_{blue}\}$,



where is $\mathcal{B}$ separated by a suitable hypersurface γ.



$J^+(\gamma) \cap \mathcal{B}$ We use the weighted vector field

$$S = |u|^p \partial_u + v^p \partial_v.$$

With the pointwise decay of Ω^2 we can then estimate the bulk by the energy flux along a constant u- and v-slice.

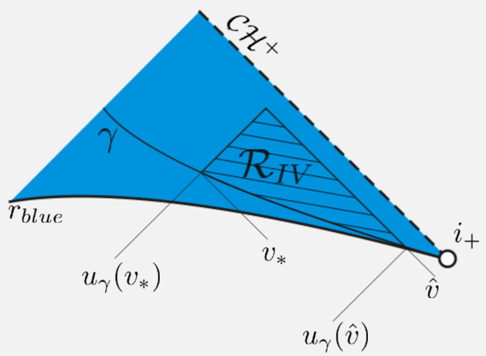
$$\Rightarrow \int_{\mathcal{R}_{IV}} |\tilde{K}^S| \mathrm{dVol} \leq \delta_1 \sup_{u_\gamma(\hat{v}) \leq \bar{u} \leq u_\gamma(v_*)} \int_{\{v_\gamma(\bar{u}) \leq v \leq \hat{v}\}} J^S_\mu(\phi) n^\mu_{u=\bar{u}} \mathrm{dVol}_{u=\bar{u}}$$

$$+ \delta_2 \sup_{v_* \leq \bar{v} \leq \hat{v}} \int_{\{u_\gamma(\hat{v}) \leq u \leq u_\gamma(\bar{v})\}} J^S_\mu(\phi) n^\mu_{v=\bar{v}} \mathrm{dVol}_{v=\bar{v}},$$

where δ_1 and δ_2 are positive constants, with $\delta_1 \to 0$ and $\delta_2 \to 0$ as $v_* \to \infty$.

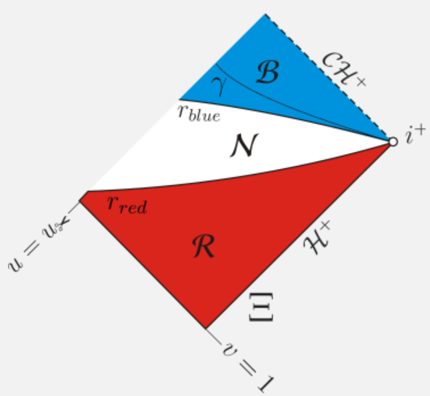
Blueshift region $\mathcal{B} = \{r_- \leq r \leq r_{blue}\}$,

where is $\mathcal{B}$ separated by a suitable hypersurface γ.



$J^+(\gamma) \cap \mathcal{B}$

$$\Rightarrow \int_{\mathbb{S}^2} \int_{u_\gamma(\hat{v})}^{u_\gamma(v_*)} J^S_\mu n^\mu_{v=\hat{v}} \mathrm{dVol}_{v=\hat{v}} \mathrm{d}\sigma^2_{\mathbb{S}} \quad \leq \quad C v_*^{-1-2\delta+p}.$$



For all of Ξ with $v_* > 1$, we obtain

$$\Rightarrow \int_{\mathbb{S}^2} \int_{v_*}^{\hat{v}} J^S_\mu n^\mu_{u=\tilde{u}} \mathrm{dVol}_{u=\tilde{u}} \mathrm{d}\sigma^2_{\mathbb{S}} \quad \leq \quad C v_*^{-1-2\delta+p}.$$

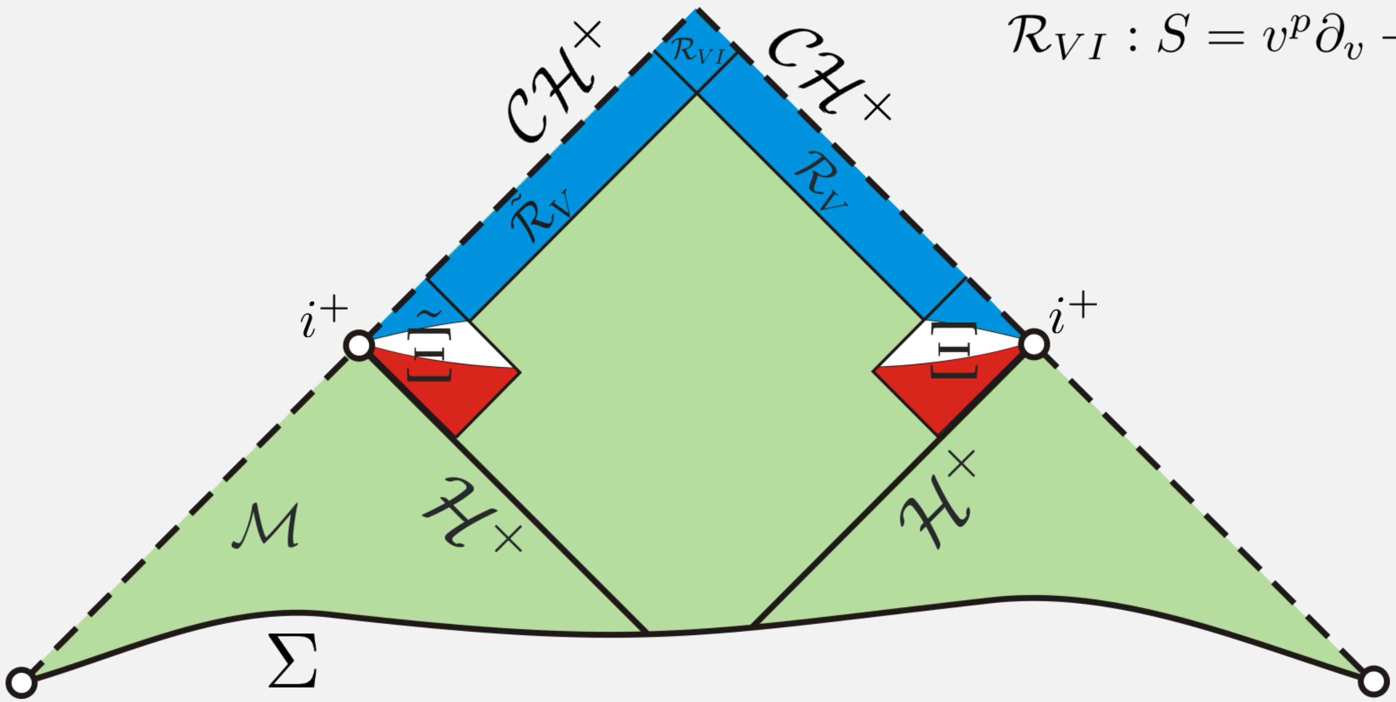
Once the Energy Theorem is obtained in the vicinity of i^+ it is straight forward to extend it to the other regions.

$\tilde{\Xi}$: substitute $u \leftrightarrow v$,
repeat all steps of Ξ

$\mathcal{R}_V : W = v^p \partial_v + \partial_u$

$\tilde{\mathcal{R}}_V : Z = \partial_v + u^p \partial_u$

$\mathcal{R}_{VI} : S = v^p \partial_v + u^p \partial_u$



Main results

<u>Energy Theorem</u>

On subextremal Reissner-Nordström spacetime $(\mathcal{M}, g)$, with mass M and charge e and $M > |e| \neq 0$, let ϕ be a solution of the wave equation $\Box_g \phi = 0$ arising from sufficiently regular Cauchy data on a two-ended asymptotically flat Cauchy surface Σ. Then for all values of Eddington-Finkelstein coordinates (u_{fix}, v_{fix}) in the black hole interior

$$\int_{\mathbb{S}^2} \int_{v_{fix}}^{\infty} \left[v^p (\partial_v \phi)^2 (u_{fix}, v) + |\nabla\!\!\!/ \phi|^2 (u_{fix}, v) \right] \mathrm{d}v \mathrm{d}\sigma_{\mathbb{S}}^2 \quad \leq \quad E, \qquad for \quad v_{fix} \geq 1,$$

$$\int_{\mathbb{S}^2} \int_{u_{fix}}^{\infty} \left[u^p (\partial_u \phi)^2 (u, v_{fix}) + |\nabla\!\!\!/ \phi|^2 (u, v_{fix}) \right] \mathrm{d}u \mathrm{d}\sigma_{\mathbb{S}}^2 \quad \leq \quad E, \qquad for \quad u_{fix} \geq 1.$$

Pointwise boundedness

The generators of spherical symmetry Ω_i, $i = 1, 2, 3$ are given by

$$\begin{aligned} \Omega_1 &= \sin\varphi\partial_\theta + \cot\theta\cos\varphi\partial_\varphi, \\ \Omega_2 &= -\cos\varphi\partial_\theta + \cot\theta\sin\varphi\partial_\varphi, \\ \Omega_3 &= -\partial_\varphi, \end{aligned}$$

and satisfy $\Box_g \Omega_i \phi = 0$.

The Main Theorem will follow from the global higher order Energy Theorem and applying Sobolev embedding.

Main results

Global higher order Energy Theorem

On subextremal Reissner-Nordström spacetime $(\mathcal{M}, g)$, with mass M and charge e and $M > |e| \neq 0$, let ϕ be a solution of the wave equation $\Box_g \phi = 0$ arising from sufficiently regular Cauchy data on a two-ended asymptotically flat Cauchy surface Σ. Then, for $v_{fix} \geq v_{✂}$, $u_{fix} > -\infty$

$$\int_{\mathbb{S}^2} \int_{v_{fix}}^{\infty} \left[(|v|+1)^p (\partial_v \Omega^k \phi)^2 (u_{fix}, v, \theta, \varphi) + \Omega^2 |\nabla\!\!\!/ \Omega^k \phi|^2 (u_{fix}, v, \theta, \varphi) \right] r^2 \mathrm{d}v \mathrm{d}\sigma_{\mathbb{S}^2} \leq E_k,$$

and for $u_{fix} \geq u_{✂}$, $v_{fix} > -\infty$

$$\int_{\mathbb{S}^2} \int_{u_{fix}}^{\infty} \left[(|u|+1)^p (\partial_u \Omega^k \phi)^2 (u, v_{fix}, \theta, \varphi) + \Omega^2 |\nabla\!\!\!/ \Omega^k \phi|^2 (u, v_{fix}, \theta, \varphi) \right] r^2 \mathrm{d}u \mathrm{d}\sigma_{\mathbb{S}^2} \leq E_k,$$

for all $k \in \mathbb{N}^0$ and $1 < p \leq 1 + 2\delta$.

Pointwise boundedness

Finally we apply Sobolev embedding on the standard spheres

$$\sup_{\{\theta,\varphi\}\in\mathbb{S}^2} |\phi(u,v,\theta,\varphi)|^2 \leq \tilde{C} \sum_{k=0}^{2} \int_{\mathbb{S}^2} \left(\Omega^k \phi\right)^2 ,$$

with

$$\sum_{k=0}^{2} \left(\Omega^k \phi\right)^2 = |\phi|^2 + \sum_{i=1}^{3} \left(\Omega_i \phi\right)^2 + \sum_{i=1}^{3} \sum_{j=1}^{3} \left(\Omega_i \Omega_j \phi\right)^2 ,$$

where k has to be understood as the order of an exponent and not as an index.

Pointwise boundedness

By the fundamental theorem of calculus and the Cauchy-Schwarz inequality it follows for all $v_* > 1, \hat{v} > v_*$ and $u \in (-\infty, u_{\scissors})$ that

$$
\begin{aligned}
\int_{\mathbb{S}^2} (\mho^k \phi)^2(u,\hat{v}) \mathrm{d}\sigma_{\mathbb{S}^2} &\leq \tilde{C} \left[\int_{\mathbb{S}^2} \left(\int_{v_*}^{\hat{v}} v^p (\partial_v \mho^k \phi)^2 (u,v) \mathrm{d}v \int_{v_*}^{\hat{v}} v^{-p} \mathrm{d}v \right) r^2 \mathrm{d}\sigma_{\mathbb{S}^2} \right. \\
&\qquad \left. + \int_{\mathbb{S}^2} (\mho^k \phi)^2 (u, v_*) \mathrm{d}\sigma_{\mathbb{S}^2} \right], \\
&\leq \tilde{C} \left[\tilde{\tilde{C}} E_k + \mathrm{data} \right].
\end{aligned}
$$

Similarly, we also integrate in u direction

$$
\int_{\mathbb{S}^2} (\mho^k \phi)^2(\hat{u}, v) \mathrm{d}\sigma_{\mathbb{S}^2} \leq \tilde{C} \left[\tilde{\tilde{C}} E_k + \mathrm{data} \right],
$$

where $u_* \geq u_{\scissors}$, $\hat{u} \in (u_*, \infty)$ and $v \in (1, \infty)$ and $k \in \mathbb{N}^0$.

Pointwise boundedness

Adding all up, we derive pointwise boundedness

$$\begin{aligned}\sup_{\{\theta,\varphi\}\in\mathbb{S}^2} |\phi(\hat{u},v,\theta,\varphi)|^2 &\leq \tilde{C}\left[\int_{\mathbb{S}^2}(\phi)^2(\hat{u},v)\mathrm{d}\sigma_{\mathbb{S}^2} + \int_{\mathbb{S}^2}(\Omega\phi)^2(\hat{u},v)\mathrm{d}\sigma_{\mathbb{S}^2} + \int_{\mathbb{S}^2}(\Omega^2\phi)^2(\hat{u},v)\mathrm{d}\sigma_{\mathbb{S}^2}\right], \\ &\leq \tilde{C}\left[\tilde{\tilde{C}}\left(E_0+E_1+E_2\right)+\mathrm{data})\right] \leq C,\end{aligned}$$

with C depending on the initial data.

The continuity statement of the Main Theorem follows easily by estimating

$$|\phi(u,v,\varphi,\theta)-\phi(\tilde{u},v,\varphi,\theta)|$$

via the fundamental theorem of calculus and Sobolev embedding, and similarly for v,φ and θ in the role of of u.

Main results

Main Theorem

On subextremal Reissner-Nordström spacetime $(\mathcal{M}, g)$, with mass M and charge e and $M > |e| \neq 0$, let ϕ be a solution of the wave equation $\Box_g \phi = 0$ arising from sufficiently regular Cauchy data on a two-ended asymptotically flat Cauchy surface Σ. Then

$$|\phi| \leq C$$

globally in the black hole interior, in particular up to and including the Cauchy horizon $\mathcal{CH}^+$.

Reference: A. F. (2014). Boundedness of massless scalar waves on Reissner-Nordström interior backgrounds. *To appear in Comm. Math. Phys. arXiv:gr-qc/1407.7093*

Latest news

"poor" linear:

- boundedness on fixed Kerr backgrounds, A. F.
- stability and extendibility of extremal RN $\mathcal{CH}^+$, Gajic
- blow up of transverse derivatives for RN, Luk & Oh

non-linear:

- *Recall:* inextendibility of spherically symmetric Einstein-Maxwell-scalar field as C^1, Dafermos
- stability of Kerr $\mathcal{CH}^+$ with C^0 extendibility, Dafermos & Luk

Conclusions

The Strong Cosmic Censorship Conjecture

A possible formulation:
Generic asymptotically flat initial data for Einstein-Maxwell spacetimes have a maximal future development which is inextendible as a Lorentzian manifold with square integrable Christoffel symbols.