Modified gravity & black holes

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Introduction

- So far, **GR** seems compatible with all observations.

- Several motivations for exploring **modified gravity**
  - Quantum gravity effects
  - Understand **cosmological acceleration**
  - Explore alternative gravitational theories
  - **Testing gravity**

- Many models of **modified gravity**

- In this talk: **scalar-tensor** theories (allowing for **2nd order derivatives** in their Lagrangian)

Higher order scalar-tensor theories

- Traditional scalar-tensor theories : $\mathcal{L}(\nabla_\lambda\phi, \phi)$

- **Generalized theories** with second order derivatives

$$\mathcal{L}(\nabla_\mu\nabla_\nu\phi,\ \nabla_\lambda\phi,\ \phi)$$

- In general, they contain an **extra degree of freedom**, expected to lead to **instabilities**

$$L(\ddot{q}, \dot{q}, q)$$

- But this can be avoided if the theory is **degenerate** (2nd-order equations of motion not necessary !)

Degenerate theories

- In general, higher-order theories with a Lagrangian of the form $L(\ddot{q}, \dot{q}, q)$ contain an **extra dof.**

- **Degenerate** theories: EOM higher order, **no extra dof**

$$L = \frac{1}{2} a\, \ddot{\phi}^2 + b\, \ddot{\phi}\, \dot{q} + \frac{1}{2} c\, \dot{q}^2 + \frac{1}{2} \dot{\phi}^2 - V(\phi, q)$$

$$\begin{cases} ac - b^2 \neq 0 & : \textbf{3 dof} \\ ac - b^2 = 0 & : \textbf{2 dof} \end{cases}$$

$$\Longrightarrow \quad L_{\rm deg} = \frac{1}{2} c \left(\dot{q} + \frac{b}{c} \ddot{\phi} \right)^2 + \frac{1}{2} \dot{\phi}^2 - V(\phi, q) \qquad \left[x \equiv q + \frac{b}{c} \dot{\phi} \quad \text{and} \quad \phi \right]$$

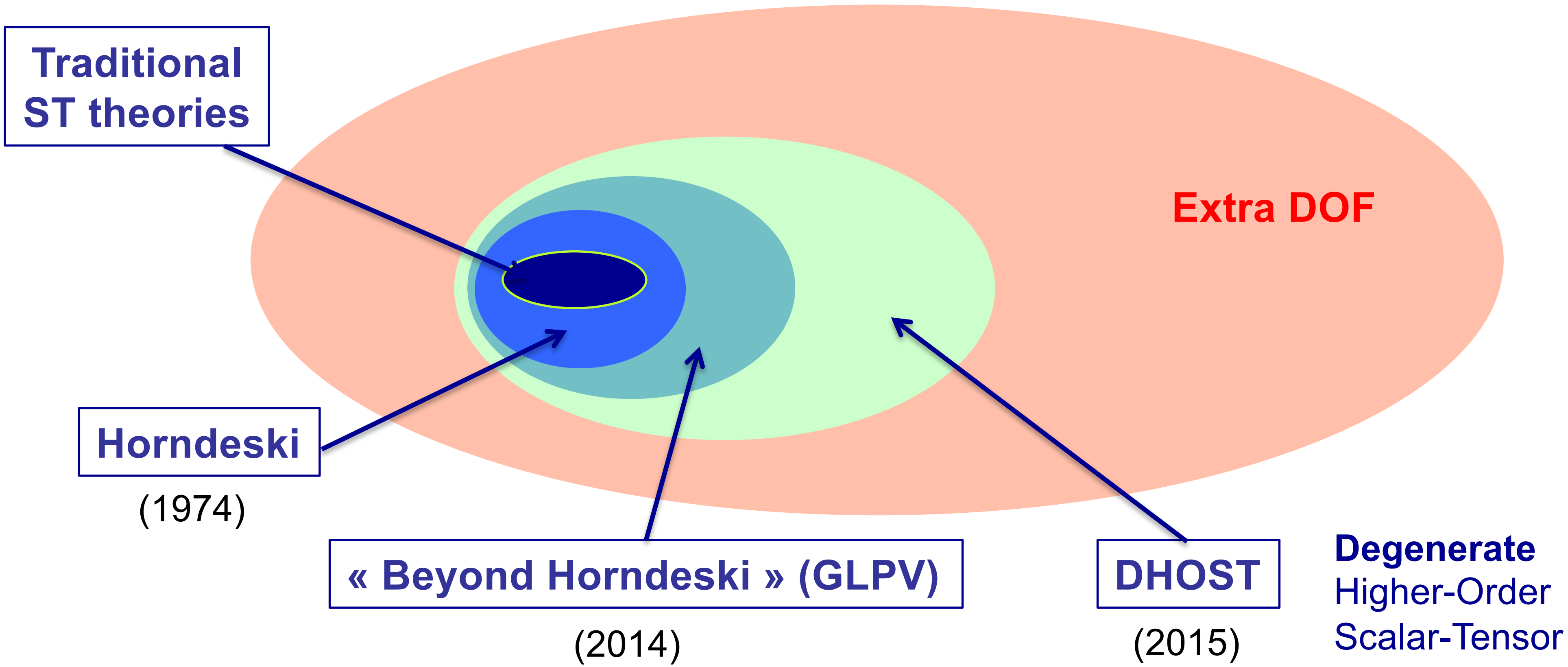
- **DHOST** (Degenerate Higher-Order Scalar-Tensor)

$$\begin{array}{ccc} \phi(t) & \longrightarrow & \phi(x^\mu) \\ q(t) & \longrightarrow & g_{\mu\nu} \end{array}$$

DL & Noui ‘15

Scalar-tensor lanscape

- Traditional theories: $\mathcal{L}(\nabla_\lambda \phi, \phi)$
- Generalized theories: $\mathcal{L}(\nabla_\mu \nabla_\nu \phi,\ \nabla_\lambda \phi,\ \phi)$
  DHOST: most general family with a **single scalar DOF**



DHOST theories

- Action of **quadratic DHOST** [DL & Noui '15]

$$S = \int d^4x \sqrt{-g} \left[P(X,\phi) + Q(X,\phi)\,\Box\phi + F(X,\phi)\,R + \sum_{i=1}^{5} A_i(X,\phi)\, L_i^{(2)} \right]$$

$$L_1^{(2)} = \phi_{\mu\nu}\,\phi^{\mu\nu}\,, \quad L_2^{(2)} = (\Box\phi)^2\,, \quad L_3^{(2)} = (\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu$$
$$L_4^{(2)} = \phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu\,, \quad L_5^{(2)} = (\phi^\mu\phi_{\mu\nu}\phi^\nu)^2$$

$$X \equiv -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$$
$$\phi_\mu \equiv \nabla_\mu\phi$$
$$\phi_{\mu\nu} \equiv \nabla_\nu\nabla_\mu\phi$$

The functions F and A_I satisfy three **degeneracy conditions**.

- Extension to **cubic order** (in $\phi_{\mu\nu}$) [Ben Achour, Crisostomi, Koyama, DL, Noui & Tasinato '16]

$$L^{(3)} = F_3(X,\phi)G_{\mu\nu}\phi^{\mu\nu} + \sum_{i=1}^{10} B_i(X,\phi)L_i^{(3)}$$

- DHOST includes Horndeski, Beyond Horndeski, Einstein-scalar-Gauss-Bonnet

  e.g. (quadratic) Horndeski: $A_2 = -A_1 = F_{,X}\,, \quad A_3 = A_4 = A_5 = 0$

Disformal transformations

[Zumalacarregui & Garcia-Bellido ‘13; Ben Achour, DL & Noui ’15]

- Transformation

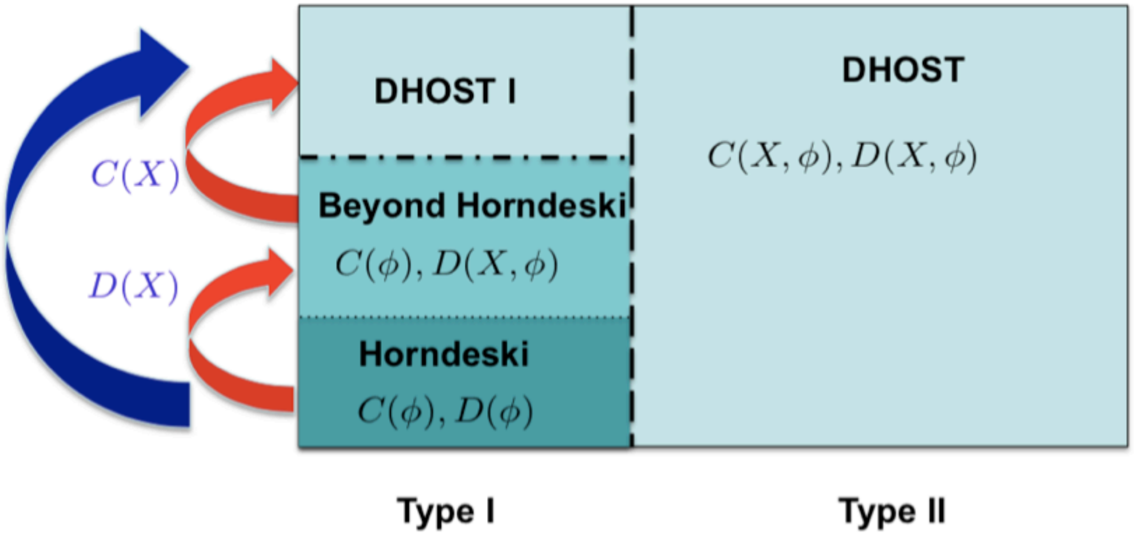
$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(X,\phi)\, g_{\mu\nu} + D(X,\phi)\, \partial_\mu\phi\, \partial_\nu\phi$$

- From $\tilde{S}\,[\phi, \tilde{g}_{\mu\nu}]$, one gets the new action

[Bekenstein ‘92]

$$S[\phi, g_{\mu\nu}] \equiv \tilde{S}\,[\phi, \tilde{g}_{\mu\nu} = C\, g_{\mu\nu} + D\, \phi_\mu\phi_\nu]$$

- DHOST families are **closed** under these transformations



- When **standard fields** are (minimally) included, two disformally related theories are **physically inequivalent** !

$$S[g_{\mu\nu}, \phi] + S_m[\Psi_m, g_{\mu\nu}] \quad \neq \quad \tilde{S}[\tilde{g}_{\mu\nu}, \phi] + S_m[\Psi_m, \tilde{g}_{\mu\nu}]$$

Black hole solutions

Non-rotating black holes

- Static spherically symmetric BH with a **nontrivial** scalar field

  – Metric: $$\mathrm{d}s^2 = -\mathcal{A}(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\mathcal{B}(r)} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)$$

  – Scalar field: $$\phi(t,r) = q\,t + \psi(r)$$ [Babichev & Charmousis ‘13]

  [$q \neq 0$ possible in **shift-symmetric** theories]

- **Examples:**

  – **« stealth » Schwarzschild:** $$\mathcal{A} = \mathcal{B} = 1 - \frac{\mu}{r}$$

  – **« BCL »** [Babichev, Charmousis & Lehébel ‘17] $$\mathcal{A} = \mathcal{B} = 1 - \frac{\mu}{r} - \xi\frac{\mu^2}{2r^2}$$

  – **« 4d Gauss-Bonnet »** [Glavan & Lin ‘19, Lu & Pang ‘20] $$\mathcal{A} = \mathcal{B} = 1 - \frac{2\mu/r}{1 + \sqrt{1 + 4\alpha\mu/r^3}}$$

  – **Scalar-Gauss-Bonnet** [Julié & Berti ‘19] $$\mathcal{A} = \mathcal{B} = 1 - \frac{\mu}{r} + a_2(r)\varepsilon^2 + \dots$$

BH with primary hair

[Bakopoulos et al. ’23; Baake et al. 23’]

- For a subfamily of DHOST theories

$$P(X) = -\frac{2\alpha}{\lambda^2} X^p \,, \;\; F(X) = 1 - 2XA_1 \,, \;\; A_1(X) = \frac{\alpha}{2} X^{p-1} \,, \;\; A_3(X) = \frac{\alpha}{2}(2p-1) X^{p-2}$$

- One can find solutions of the form

$$\mathrm{d}s^2 = -\mathcal{A}(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\mathcal{B}(r)} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right) \qquad \phi(t,r) = q\,t + \psi(r)$$

with $\displaystyle \mathcal{A}(r) = 1 - \frac{2\mu}{r} - \frac{2\xi_p}{r}\int_0^r \frac{u^2\mathrm{d}u}{(1+u^2)^p}$ $\qquad \displaystyle \xi_p \equiv \alpha(2p-1)\left(\frac{q^2}{2}\right)^p$

and $\displaystyle X = \frac{q^2}{2(1+r^2)}, \qquad \psi'(r) = \frac{q}{\mathcal{A}(r)}\sqrt{1 - \frac{\mathcal{A}(r)}{1+r^2}}$

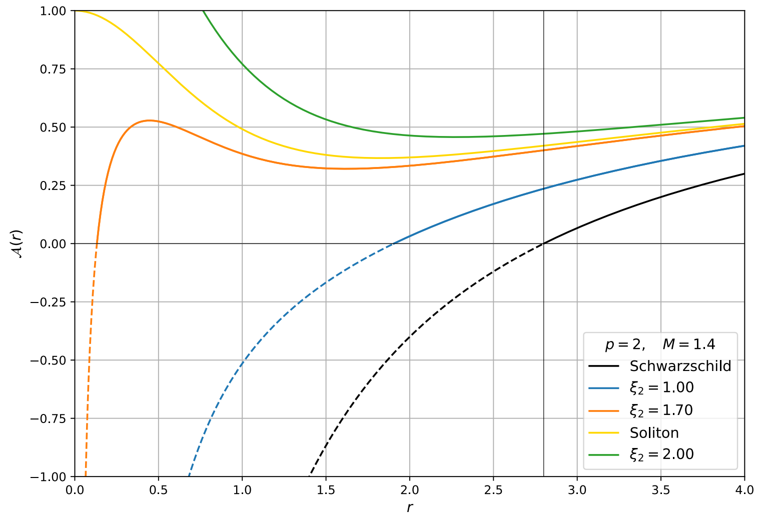
BH with primary hair

- Examples:
  - For $p = 1/2$: « stealth » Schwarzschild (Horndeski theory)
  - For $p = 2$: deformation of Schwarzschild

$$\mathcal{A}(r) = 1 - \frac{2M}{r} + \xi_2 \left(\frac{\pi/2 - \arctan r}{r} + \frac{1}{1+r^2} \right)$$

  - No singularity in r =0 if

$$M = \frac{\pi}{4}\xi_2$$



One finds **regular BHs**

$$M > M_{\rm BH}^{\rm reg} \simeq 2.21 \quad (\text{i.e. } \xi_2 > \xi_{2,\rm BH}^{\rm reg} \simeq 2.82)$$

New BHs via disformal transformations

[Charmousis, Iteanu, DL & Noui 25']

- New «frame», e.g. such that $\tilde{A}_1 = 0$, which implies $c_g = c$

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(X)g_{\mu\nu} + D(X)\partial_\mu\phi\,\partial_\nu\phi$$

- New metric ($p = 2$)

$$\mathrm{d}\tilde{s}^2 = -\tilde{\mathcal{A}}\;\mathrm{d}t_*^2 + \tilde{\mathcal{A}}^{-1}\left(1 - \frac{\xi_2}{3(1+r^2)^2}\right)\mathrm{d}r^2 + r^2 d\Omega^2$$

with $\tilde{\mathcal{A}} = \mathcal{A} - \dfrac{\xi_2}{3(1+r^2)}$

$$t_* = t - q\int \frac{D\psi'}{C\mathcal{A} - Dq^2}\mathrm{d}r$$

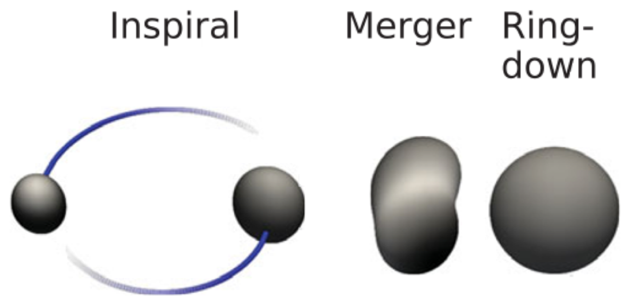
  - New time coordinate:
  - Nature of the metric can differ
  - Shift of the horizon (if BHs)

- Other choices of frames (e.g. Horndeski: $\tilde{A}_1 = -\tilde{F}_{\tilde{X}}$)

Black hole perturbations

Black hole perturbations

- **GW astronomy** provides a new window to **test GR,** in particular in the strong field regime.



- **Ringdown phase** of a BH merger
  is interesting for modified gravity models,
  as it can be described by BH perturbations.

- **Deviations** in the context of **DHOST** theories ?

Black hole perturbations

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{bgd}} + h_{\mu\nu}$$

- In the frequency domain: $f(t,r) = f(r)\, e^{-i\omega t}$

- **Axial** (or odd-parity) modes: $h_0(r), h_1(r)$ [Regge-Wheeler gauge]

$$h_{\mu\nu} = \sum_{\ell,m} \begin{pmatrix} 0 & 0 & \frac{1}{\sin\theta} h_0^{\ell m} \partial_\varphi & -\sin\theta h_0^{\ell m} \partial_\theta \\ 0 & 0 & \frac{1}{\sin\theta} h_1^{\ell m} \partial_\varphi & -\sin\theta h_1^{\ell m} \partial_\theta \\ \mathrm{sym} & \mathrm{sym} & 0 & 0 \\ \mathrm{sym} & \mathrm{sym} & 0 & 0 \end{pmatrix} Y_{\ell m}(\theta,\varphi)$$

- **Polar** (or even-parity) modes: H_0, H_1, H_2, K (and $\delta\phi$)

$$h_{\mu\nu} = \sum_{\ell,m} \begin{pmatrix} A(r) H_0^{\ell m}(r) & H_1^{\ell m}(r) & 0 & 0 \\ H_1^{\ell m}(r) & A^{-1}(r) H_2^{\ell m}(r) & 0 & 0 \\ 0 & 0 & K^{\ell m}(r)\, r^2 & 0 \\ 0 & 0 & 0 & K^{\ell m}(r)\, r^2 \sin^2\theta \end{pmatrix} Y_{\ell m}(\theta,\varphi)$$

Axial modes in GR

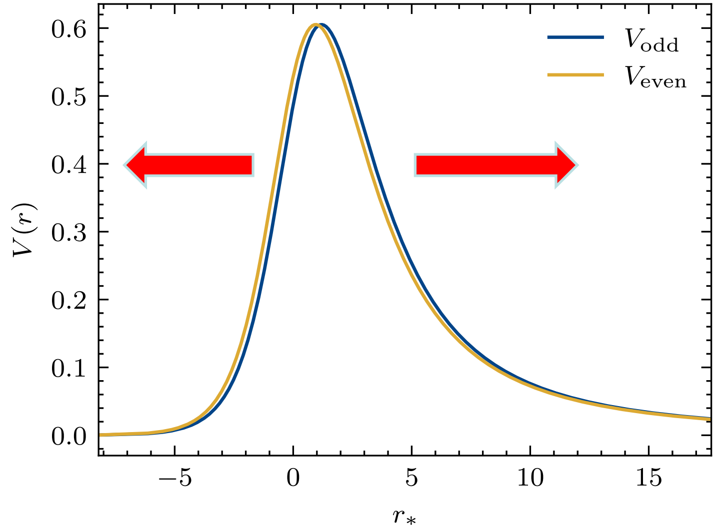
- The linearised metric eqs yield only 2 independent eqs

$$\frac{dY}{dr} = M(r)\,Y(r)\,, \quad Y = \begin{pmatrix} h_0 \\ h_1/\omega \end{pmatrix}$$

or, in a **Schroedinger** form, [Regge & Wheeler '57]

$$\frac{d^2\hat{Y}}{dr_*^2} + \left(\omega^2 - V(r)\right)\hat{Y} = 0$$

[r_* tortoise coordinate]



- **Asymptotically** ($r_* \to -\infty, +\infty$)

$$e^{-i\omega t}\,\hat{Y}(r) \approx a_+\,e^{-i\omega(t-r_*)} + a_-\,e^{-i\omega(t+r_*)}$$

outgoing ingoing

- **Quasi-normal modes**: $a_+^{\rm hor} = 0 \quad \text{and} \quad a_-^{\infty} = 0$

Axial modes in DHOST

[DL, Noui & Roussille '22]

- The equations have a similar structure

$$\frac{dY}{dr} = MY\,, \qquad M \equiv \begin{pmatrix} 2/r + i\omega\Psi & -i\omega^2 + 2i\lambda\Phi/r^2 \\ -i\Gamma & \Delta + i\omega\Psi \end{pmatrix}$$

$$\lambda \equiv \frac{\ell(\ell+1)}{2} - 1$$

where Ψ, Φ, Γ and Δ depend on the Lagrangian's functions and on the background.

- **Correspondence (in quadratic DHOST theories)**

DHOST axial modes in $g_{\mu\nu}$ $\longleftrightarrow$ **GR** axial modes in $\hat{g}_{\mu\nu}$

with the **effective metric**

$$d\hat{s}^2 = \hat{g}_{\mu\nu}dx^\mu dx^\nu = |\mathcal{F}|\sqrt{\frac{\Gamma\mathcal{B}}{\mathcal{A}}}\left(-\Phi\,(dt - \Psi dr)^2 + \Gamma\Phi dr^2 + r^2 d\Omega^2\right)$$

Effective metric for axial modes

[Charmousis, Iteanu, DL & Noui 25']

- For **BHs with primary hair**, the effective metric reduces to

$$d\hat{s}^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu = \sqrt{F}\left(-\Phi\, dt_*^2 + \frac{F}{\Phi} dr^2 + r^2 d\Omega^2\right)$$

with $\Phi = \mathcal{A} - q^2 A_1$

- In **quadratic** DHOST theories, the **effective metric** corresponds to a disformal transformation such that $\hat{F} = 1$ and $\hat{A}_1 = 0$.

- In our case, $\hat{g}_{\mu\nu} = \sqrt{F}\,(g_{\mu\nu} + A_1\, \phi_\mu \phi_\nu)$

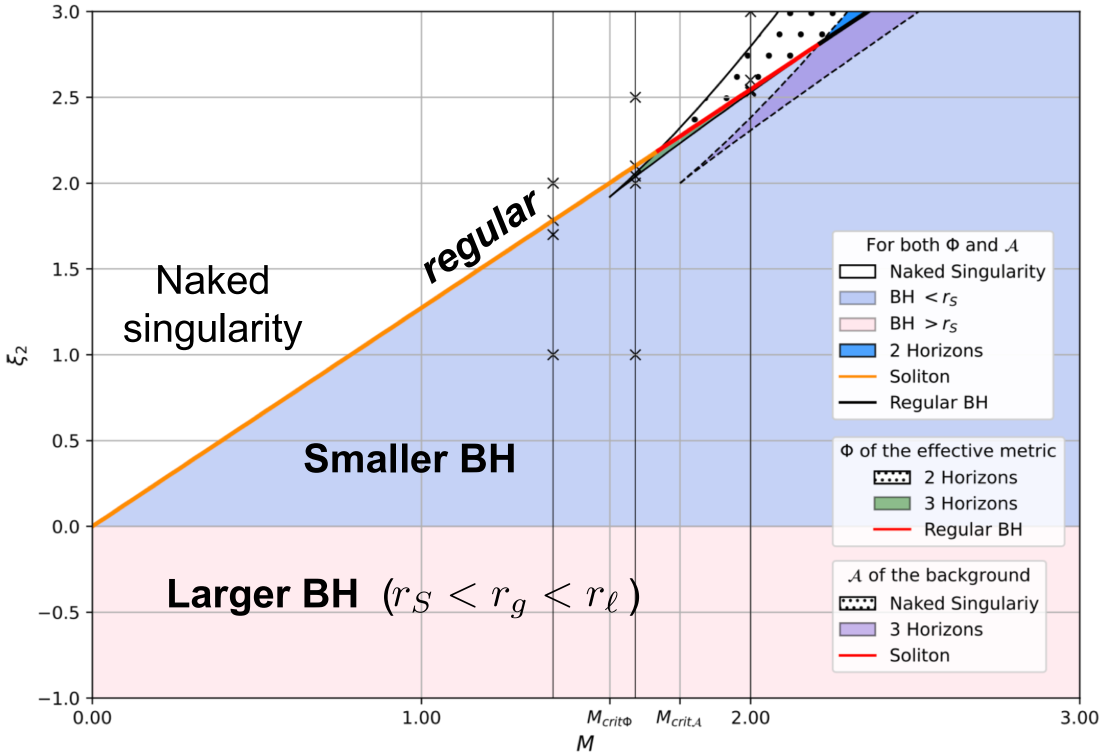
- Photons & gravitons **«see» different geometries**: $g_{\mu\nu}$ and $\hat{g}_{\mu\nu}$, and therefore different horizons (for BHs), determined by

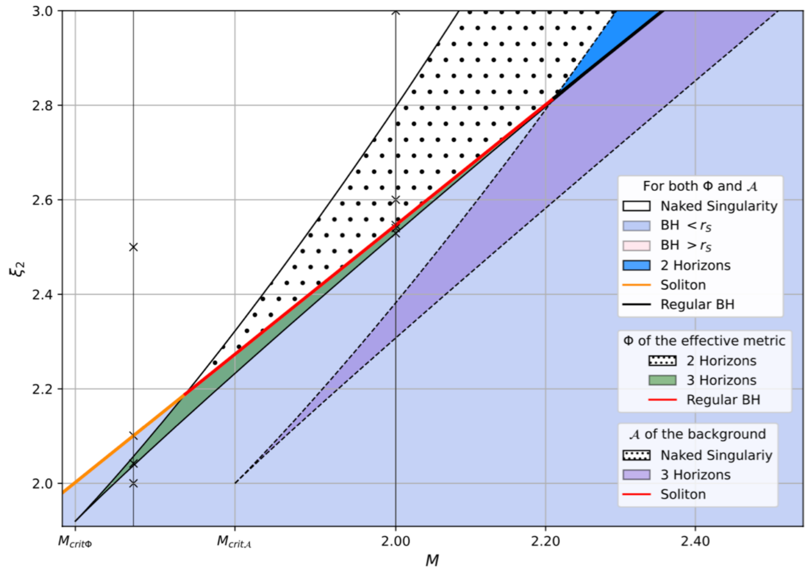
$$\mathcal{A}(r_\ell) = 0\,, \qquad \Phi(r_g) = 0$$

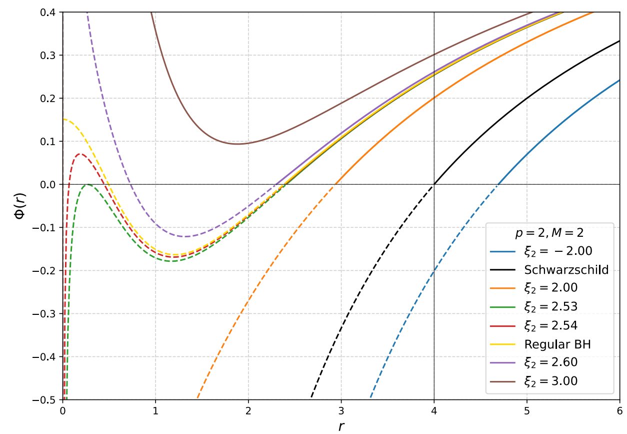
Effective metric for axial modes

[Charmousis, Iteanu, DL & Noui 25']

- Phase diagram for the background and effective metrics







For $\xi_2 > 0$: $r_\ell < r_g < r_S$

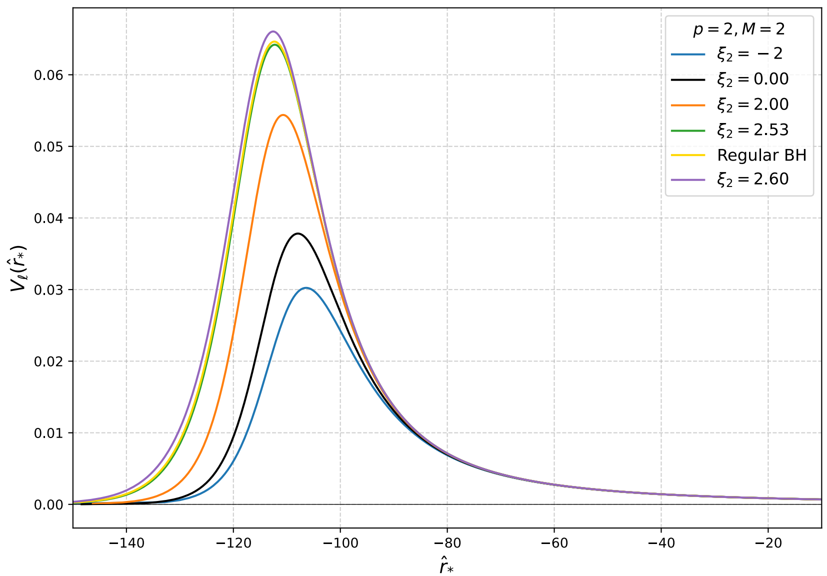
Axial quasi-normal modes

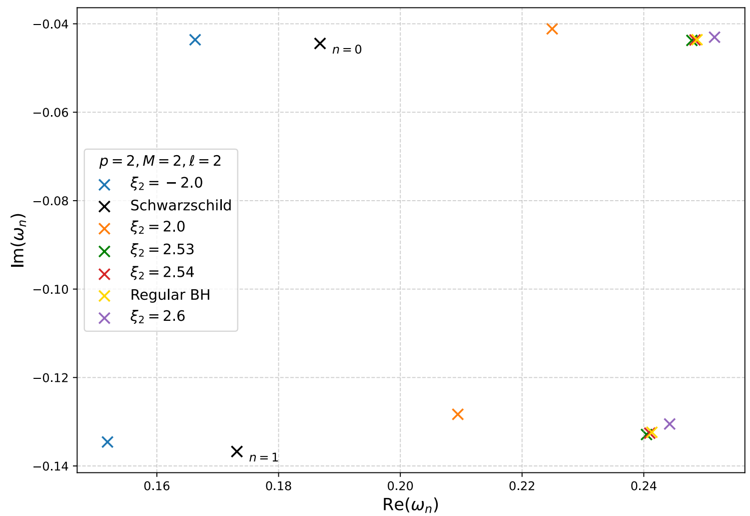
- As in GR, one can write a **Schroedinger-like** equation

$$-\frac{d^2\mathcal{Y}}{d\hat{r}_*^2} + V_\ell\,\mathcal{Y} = \omega^2\,\mathcal{Y} \qquad \mathcal{Y} = \frac{\Phi}{rF^{1/4}\omega}(h_1 + \Psi h_0)$$

where

$$V_\ell(r) = \Phi\left[\frac{\ell^2+\ell-2}{r^2} - \frac{1}{r}\kappa_1(F,F',r)\Phi' + \frac{2}{r^2}\kappa_2(F,F',F'',r)\Phi\right]$$





Polar modes

- The linearised metric equations yield
  - 2 independent equations in GR (1 dof)
  - **4 independent equations** in DHOST theories (2 dof)

- In **GR**: 2-dimensional system $Y' = M\,Y$, which can be written in a Schroedinger form. [Zerilli '70]

- In **DHOST**, the system $Y' = M\,Y$ is now 4-dimensional, with

$$Y = {}^{T}(K\ \delta\phi\ H_1\ H_0)$$

- It is convenient to do an **asymptotic analysis** of the first-order system.

Asymptotics of a diffential system

- Instead of a Schroedinger-like approach, one can use directly the initial first-order equations of motion and their asymptotic limit:

$$\frac{dY}{dz} = M(z)\,Y\,, \quad M(z) = M_r z^r + M_{r-1} z^{r-1} + \ldots \quad (z \to \infty)$$

- The generic solution is of the form

$$Y(z) = e^{\boldsymbol{\Upsilon}(z)}\, z^{\boldsymbol{\Delta}}\, \mathbf{F}(z)\, Y_0, \qquad (z \to \infty)$$

- There exists a well-defined algorithm to determine the diagonal matrices $\boldsymbol{\Upsilon}(z)$ and $\boldsymbol{\Delta}$. [Balser '99]

Idea: diagonalise, order by order, the matrix *M*, with $Y(z) = P(z)\,\tilde{Y}(z)$

$$\frac{d\tilde{Y}}{dz} = \tilde{M}(z)\,\tilde{Y}, \qquad \tilde{M}(z) \equiv P^{-1} M P - P^{-1}\frac{dP}{dz}$$

Numerical polar modes

- Study the **asymptotic behaviour** of the 4-dim system at spatial infinity and near the horizon, and **extract the asymptotic independent modes**.

- At spatial infinity, one can identify
  - 2 « gravitational » modes
  - 2 « scalar» modes

- Similar results near the horizon

- **Starting point for a numerical determination** of the quasi-normal modes of these BHs in modified gravity.

Conclusion

- **DHOST theories:** most general framework for scalar-tensor theories propagating a single scalar dof.

- DHOST theories give a **very rich phenomenology**, which can **describe deviations** from General Relativity, both for compact objects and in cosmology, and be tested in future observations.

- Interesting **subfamily** of DHOST theories allowing for **exact BHs** solutions with **primary hair**:
  – Axial modes: effective metric & QNMs
  – Polar modes: for future work