Black holes with electroweak hair

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Romain Gervalle and M.S.V.
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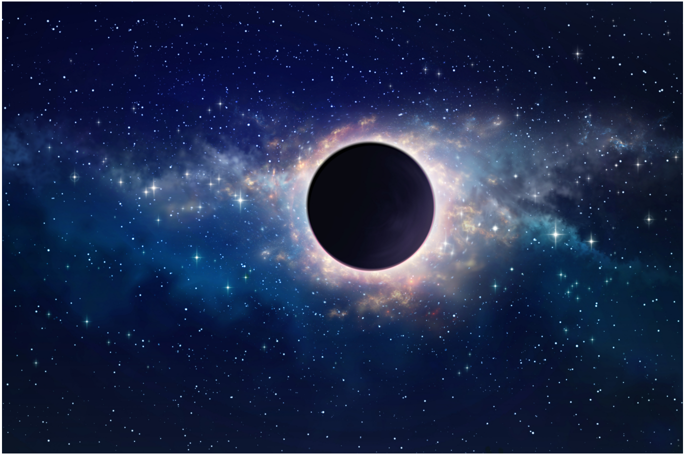
Isolated black hole – Kerr-Newman geometry



3 parameters: mass M, charge Q, angular momentum J.

Brief history: no-hair conjecture

No-hair conjecture /Ruffini and Wheeler, 1971/: black holes formed by gravitational collapse are characterized by their mass, angular momentum, and electric/magnetic charge. These are the only parameters that can survive during the gravitational collapse, all other information is lost. Black holes have no memory.

No-hair conjecture

From January 1971, pages 30–41

Introducing the black hole

Remo Ruffini and John A. Wheeler

According to present cosmology, certain stars end their careers in a total gravitational collapse that transcends the ordinary laws of physics.

At the time of this article, **Remo Ruffini** and **John Wheeler** were both at Princeton University; Wheeler, on leave from Princeton, was spending a year at the California Institute of Technology and Moscow State University.

The quasi-stellar object, the pulsar, the neutron star have all come onto the scene of physics within the space of a few years. Is the next entrant destined to be the black hole? If so, it is difficult to think of any development that could be of greater significance. A black hole, whether of "ordinary size" (approximately one solar mass, 1 $M_\odot$) or much larger (around $10^6\,M_\odot$ to $10^{10}\,M_\odot$, as proposed in the nuclei of some galaxies), provides our "laboratory model" for the gravitational collapse, predicted by Einstein's theory, of the universe itself.

A black hole is what is left behind after an object has undergone complete gravitational collapse. Spacetime is so strongly curved that no light can come out, no matter can be ejected, and no measuring rod can ever survive being put in. Any kind of object that falls into the black hole loses its separate identity, preserving only its mass, charge, angular momentum, and linear momentum (see figure 1). No one has yet found a way to distinguish between two black holes constructed out of the most different kinds of matter if they have the same mass, charge, and angular momentum. Measurement of these three determinants is permitted by their effect on the Kepler orbits of test objects, charged and uncharged, in revolution about the black hole.

How the physics of a black hole looks depends more upon an act of choice by the observer himself than on anything else. Suppose he decides to follow the collapsing matter through its collapse down into the black hole. Then he will see it crushed to indefinitely high density, and he himself will be torn apart eventually by indefinitely increasing tidal forces. No restraining force whatsoever has the power to hold him away from this catastrophe, once he crossed a certain critical surface known as the "horizon." The final collapse occurs a finite time after the passage of this surface, but it is inevitable. Time and space are interchanged inside a black hole in

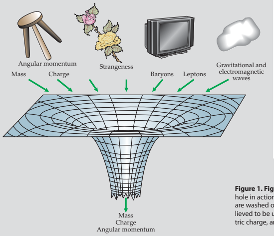


Figure 1. Figurative representation of a black hole in action. All details of the infalling matter are washed out. The final configuration is believed to be uniquely determined by mass, electric charge, and angular momentum.

Uniqueness and no-hair theorems

- Uniqueness theorems /Israel, Robinson, Mazur/: All electrovacuum holes are described by the Kerr-Newman metrics. This confirms the conjecture.
- Are there other black holes, not described by Kerr-Newman metrics ?
- No-hair theorems /Bekenstein, 1972,.../ confirm the conjecture for a number of special cases. Considering

$$G_{\mu\nu} = T_{\mu\nu}(\Phi), \qquad \Box\Phi = U(\Phi),$$

  where Φ = scalar, spinor, massive vector field, etc., field, one can show that the only black hole solutions are of the Kerr-Newman type.
- However, if $\Phi = A^a_\mu$ is a pure Yang-Mills field then there are new black holes without new charges:

Non-Abelian Einstein-Yang–Mills black holes

M. S. Volkov and D. V. Gal'tsov
M. V. Lomonosov Moscow State University

(Submitted 7 September 1989)
Pis'ma Zh. Eksp. Teor. Fiz. **50**, No. 7, 312–315 (10 October 1989)

Solutions of the self-consistent system of Einstein-Yang–Mills equations with the SU(2) group are derived to describe black holes with a non-Abelian structure of gauge fields in the external region.

In the case of the electrovacuum, the most general family of solutions describing spherically symmetric black holes is the two-parameter Reissner–Nordström family, which is characterized by a mass M and an electric charge Q. It was recently shown for the Einstein-Yang–Mills systems of equations with the SU(2) group that a corresponding assertion holds when the hold has a nonvanishing color-magnetic charge. In this case the structure of the Yang–Mills hair is effectively Abelian.[1] In the present letter we numerically construct a family of definitely non-Abelian solutions for Einstein-Yang–Mills black holes in the case of zero magnetic charge. These solutions are characterized by metrics which asymptotically approach the Schwarzschild metric far from the horizon but are otherwise distinct from metrics of the Reissner–Nordström family. In addition to the complete Schwarzschild metric, the family of solutions is parametrized by a discrete value of n: the number of nodes of the gauge function. For a

Zoo of hairy black holes

- before 2000: Einstein-Yang-Mills black holes and their generalizations – higher gauge groups, additional fields (Higgs, dilaton), non-spherical solutions, stationary generalizations, Skyrme black holes, Gauss-Bonnet, . . . /M.S.V.+Gal'tsov, Phys.Rep. 319 (1999) 1/

- after 2000: black holes via engineering the scalar field potential, Horndeski black holes, spontaneously scalarized black holes, black holes supporting spinning clouds of ultralight bosons /Herdeiro-Radu/, hairy black holes in higher dimensions, with stringy corrections, with massive gravitons /Gervalle+M.S.V., 2020/, etc, . . . /M.S.V., 1601.0823/

- Which of these solutions are physical ? Unfortunately, one cannot be too optimistic in this respect.

Present status of hairy black holes

- Almost all known hairy solutions have been obtained either within too much simplified models, or within exotic models relying on a new physics = yet undiscovered particles and fields. They are nice theoretically but their physical relevance is not obvious.

- New physics (stringy effects, SUSY, GUT fields, Horndeski fields, ultralight Dark Matter, massive gravitons, etc) may exist. However, its existence has not been confirmed yet.

- To be physically relevant, solutions should be obtained within General Relativity (GR) + Standard Model (SM) of fundamental interactions.

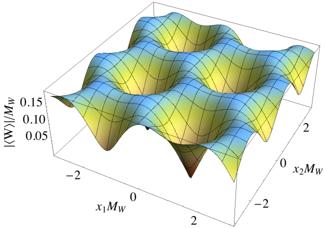
- The SM contains the QCD sector with pure Yang-Mills (gluons). Therefore, hairy black holes with Yang-Mills field may have some relevance. However, classical configurations in QCD are destroyed by large quantum corrections.

Electroweak black holes ?

- The Standard Model contains also the electroweak (EW) sector where the quantum corrections are not very large. Therefore, it makes sense to study classical solutions of the Einstein-Weinberg-Salam theory. This theory contains the Einstein-Maxwell sector and hence describes the Kerr-Newman black holes.
- Does it describe something else ?
- Only unphysical limits of the electroweak theory (vanishing Weinberg angle) have been analyzed in the black hole context, since in the full theory the spherical symmetry is lost due to the electroweak condensation.

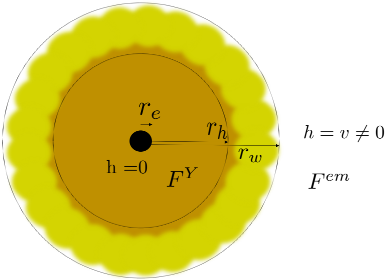
Electroweak condensation /Ambjorn-Olesen 1989/

- Constant homogeneous magnetic field $\vec{\mathcal{B}} = (0, 0, \mathcal{B})$ may exist if only $\mathcal{B} < m_{\rm w}^2/e \approx 10^{20}$ Tesla.
- For $m_{\rm w}^2/e < \mathcal{B} < m_{\rm h}^2/e$ the vacuum structure changes leading to the appearance of a condensate of massive W, Z, Φ fields forming a lattice of vortices (flux tubes). Anti-Lenz: the magnetic field is maximal where the condensate is maximal.



- For $B > m_{\rm h}^2/e$ the vortices disappear and the Higgs field approaches zero – the full electroweak symmetry is restored.

Magnetic electroweak black hole /Maldacena 2020/



Radial magnetic field near the horizon where Higgs=0, followed by electroweak "corona" made of vortex pieces, followed by radial magnetic field in the far field where Higgs is constant = magnetic Reissner-Nordstrom.

Nobody tried to confirm this

Preliminary analyzis in flat space

The electroweak corona should exist already in flat space around a pointlike magnetic charge, but the best known magnetic monopole of t' Hooft-Polyakov is not described by the Standard Model.

Electroweak theory contains two types of static, spherically symmetric monopole solutions, both with infinite energy:

- Pointlike Dirac monopole for any value of the magnetic charge $n = \pm 1, \pm 2, \ldots$. Unstable with respect to condensation.
- Non-Abelian monopole of Cho-Maison for $n = \pm 2 \Rightarrow$ superposition of a pointlike hypermagnetic U(1) monopole and a regular SU(2) condensate. Can be viewed as a stable remnant of condensation around the $n = \pm 2$ Dirac monopole.

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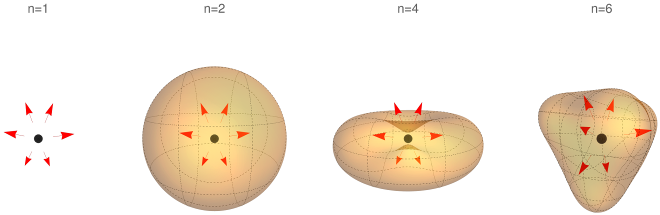
Electroweak monopoles in flat space



We constructed numerically monopoles with axial symmetry up to $|n| = 200$. They contain a pointlike magnetic charge surrounded by a condensate. The energy is infinite due to the central singularity.

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When gravity is taken into account, the singularity should be shielded by a horizon and the energy will become finite.

Including gravity

/PRL 133 (2024) 171402;
arXiv:2504.09304 /

Einstein-Weinberg-Salam theory

$$\mathcal{L} = \frac{1}{2\kappa} R + \mathcal{L}_{\rm WS}$$

$$\mathcal{L}_{\rm WS} = -\frac{1}{4g^2} \mathrm{W}^a_{\mu\nu} \mathrm{W}^{a\mu\nu} - \frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu} - (D_\mu \Phi)^\dagger D^\mu \Phi - \frac{\beta}{8} \left(\Phi^\dagger \Phi - 1 \right)^2$$

where Higgs is a complex doublet, $\Phi = (\phi_1, \phi_2)^{\rm T}$,

$$\mathrm{W}^a_{\mu\nu} = \partial_\mu \mathrm{W}^a_\nu - \partial_\nu \mathrm{W}^a_\mu + \epsilon_{abc} \mathrm{W}^b_\mu \mathrm{W}^c_\nu \,, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \,,$$

$$D_\mu \Phi = \left(\partial_\mu - \frac{i}{2} B_\mu - \frac{i}{2} \tau^a \mathrm{W}^a_\mu \right) \Phi \,.$$

The length scale and mass scale are $\boldsymbol{l}_0 = 1.5 \times 10^{-16}$ cm and $\boldsymbol{m}_0 =$ 128.6 GeV. The couplings $g' = \sin\theta_{\rm W}$, $g = \cos\theta_{\rm W}$,

$$g^2 = 0.78, \quad g'^2 = 0.22, \quad \beta = 1.88, \quad \kappa = \frac{4e^2}{\alpha} \frac{m_z^2}{M_{\rm pl}^2} = 5.30 \times 10^{-33}.$$

Electron charge $e = gg'$, $\alpha = 1/137$. The Z, W, Higgs masses in unites of $\boldsymbol{m}_0$ are $m_z = 1/\sqrt{2}$, $m_w = g m_z$, $m_h = \sqrt{\beta} m_z$.

Electromagnetic field (no unique definition if $\Phi \neq const$):

Nambu: $e\mathcal{F}_{\mu\nu} = g^2 B_{\mu\nu} - g'^2 n_a \mathrm{W}^a_{\mu\nu}, \quad n_a = (\Phi^\dagger \tau_a \Phi)/(\Phi^\dagger \Phi)$

defines conserved electric and magnetic currents

$$4\pi \mathcal{J}^\mu = \nabla_\nu \mathcal{F}^{\mu\nu}, \qquad 4\pi \tilde{\mathcal{J}}^\mu = \nabla_\nu \tilde{\mathcal{F}}^{\mu\nu},$$

magnetic charge

$$\boxed{P = \int \tilde{\mathcal{J}}^0 \sqrt{-\mathrm{g}}\, d^3x.}$$

t'Hooft: $F_{\mu\nu} = \mathcal{F}_{\mu\nu} + \epsilon_{abc} n^a \mathcal{D}_\mu n^b \mathcal{D}_\nu n^c = \partial_\mu A_\nu - \partial_\nu A_\mu,$

electric current

$$\boxed{4\pi J^\mu = \nabla_\nu F^{\mu\nu}}.$$

Using Nambu for magnetic charge and t'Hooft for electric current.

30 coupled equations to solve:

Weinberg-Salam:

$$\nabla^{\mu} B_{\mu\nu} = g'^2 \, \frac{i}{2} \, (\Phi^{\dagger} D_{\nu} \Phi - (D_{\nu}\Phi)^{\dagger} \Phi),$$

$$\mathcal{D}^{\mu} \mathrm{W}^{a}_{\mu\nu} = g^2 \, \frac{i}{2} \, (\Phi^{\dagger} \tau^{a} D_{\nu} \Phi - (D_{\nu}\Phi)^{\dagger} \tau^{a} \Phi),$$

$$D_{\mu} D^{\mu} \Phi - \frac{\beta}{4} \, (\Phi^{\dagger}\Phi - 1)\Phi = 0,$$

Einstein:

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$ where $\kappa \sim 10^{-33}$ is very small and

$$T_{\mu\nu} = \frac{1}{g^2} \mathrm{W}^{a}{}_{\mu\sigma} \mathrm{W}^{a}{}_{\nu}{}^{\sigma} + \frac{1}{g'^2} B_{\mu\sigma} B_{\nu}{}^{\sigma} + 2 D_{(\mu} \Phi^{\dagger} D_{\nu)} \Phi + \mathrm{g}_{\mu\nu} \mathcal{L}_{\mathrm{WS}}$$

=30 coupled equations. Vacuum solution:

$$\mathrm{g}_{\mu\nu} = \eta_{\mu\nu}, \quad \mathrm{B} = \mathrm{W} = 0, \;\; \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Simplest solutions: Reissner-Nordstrom, RN-de Sitter

Same electroweak fields as for Dirac monopole, *Higgs in vacuum:*

$$B = W^3 = -\frac{n}{2}(\cos\vartheta \mp 1)\, d\varphi, \quad W^1 = W^2 = 0, \quad \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$A = \frac{g}{g'} B + \frac{g'}{g} W^3 = \frac{B}{e}, \quad \vec{\nabla} \wedge \vec{A} = \frac{P\vec{r}}{r^3}, \quad P = \frac{n}{2e}, \quad n \in \mathbb{Z}$$

n, P=magnetic charge; RN geometry:

$$ds^2 = -N(r)\, dt^2 + \frac{dr^2}{N(r)} + r^2 \left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right),$$

$$N(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \qquad Q^2 = \frac{\kappa n^2}{8e^2}, \qquad r_h = M + \sqrt{M^2 - Q^2}$$

Describes the $r \to \infty$ limit of the hairy black holes.
RN-de Sitter: $W^1 = W^2 = W^3 = \Phi = 0$, same B. Its extremal limit describes the horizon geometry of extremal hairy black holes.

Perturbations around Reissner-Nordström

- The horizon value of the magnetic field is $\mathcal{B}_h = P/r_h^2$ is small if r_h is large, then the solution is expected to be stable.
- If r_h is small then $\mathcal{B}_h = P/r_h^2$ is large and the solution is expected to become unstable with respect to condensation.
- Therefore, the solution is expected to change stability when r_h changes.
- When the stability starts to change, this indicates the appearance of a new family of solutions which start deviating from the Reissner-Nordström family.

Consider generic perturbations around RN,

$$g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}, \quad \mathrm{W}_\mu^a \to \mathrm{W}_\mu^a + \delta \mathrm{W}_\mu^a$$

$$B_\mu \to B_\mu + \delta B_\mu, \quad \Phi \to \Phi + \delta\Phi$$

Perturbations $w_\mu = \delta W^1_\mu + i\delta W^2_\mu$ fulfil charged Proca:

$$
\begin{aligned}
D^\mu w_{\mu\nu} + ieF_{\nu\sigma}w^\sigma &= m_{\rm w}^2 w_\nu \quad \Rightarrow \quad \text{(Newman-Penrose)} \\
w_\mu dx^\mu &= \sum_{\rm m\in[-j,j]} c_{\rm m} e^{i\omega t}\psi(r) w_{j{\rm m}}(\vartheta) e^{i{\rm m}\varphi}, \\
w_{j{\rm m}}(\vartheta) &= (\sin\vartheta)^j \left(\tan\frac{\vartheta}{2}\right)^{\rm m} (d\vartheta + i\sin\vartheta d\varphi), \\
j &= |n|/2 - 1 \;\Rightarrow\; \boxed{j = 0 \text{ only if } |n| = 2}.
\end{aligned}
$$

For $|n| > 2$ perturbations are not spherically symmetric.

$$
\left(-\frac{d^2}{dr_\star^2} + N(r)\left[m_{\rm w}^2 - \frac{|n|}{2r^2}\right]\right)\psi(r) = \omega^2\psi(r). \qquad (\star)
$$

$\omega^2 > 0$ if r_h is large, $\omega^2 < 0$ if r_h is small $\Rightarrow$ there is an intermediate value: a condensation threshold $r_h = r_h^0(n)$ for which there is a zero mode $\psi_0(r)$ describing a condensate that starts to appear.

Horizon distribution of vortices

The static condensate

$$w = \sum_{\mathrm{m}\in[-j,j]} c_{\mathrm{m}} \psi_0(r) w_{j\mathrm{m}}(\vartheta) e^{i\mathrm{m}\varphi}$$

produces a current $J^\mu = \nabla_\sigma \Im(\bar{w}^\sigma w^\mu)$ tangent to the horizon.
The current sources second order corrections for the F, Z, Φ fields forming vortices orthogonal to the horizon. The coefficients c_{m} are obtained via minimizing the condensate energy:

$$\langle |w_\mu|^4 \rangle \equiv \int |w_\mu|^4 \sqrt{-\mathrm{g}}\, d^3x \,,$$

by keeping fixed the norm

$$\langle |w_\mu|^2 \rangle \equiv \int |w_\mu|^2 \sqrt{-\mathrm{g}}\, d^3x = const.$$

This gives values of c_{m} determining positions of $|n| - 2$ vortices homogeneously distributed over the horizon:

Lattice of vortices – corona

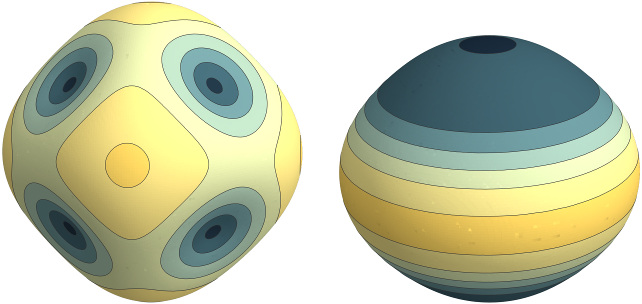


Figure: Left: the horizon distribution of the W-condensate $\bar{w}^{\mu} w_{\mu}$ corresponding to the global energy minimum for $n = 10$. The level lines coincide with the electric current flow forming loops around 8 radial vortices (dark spots) repelling each other and forming a lattice.
Right: the same when all vortices merge into two oppositely directed multi-vortices with axial symmetry, $c_{\rm m} \sim \delta_{\rm 0m}$, also a stationary point.

Non-perturbative analysis

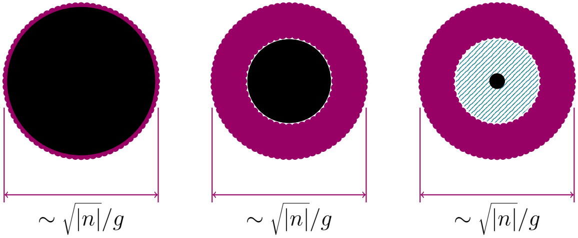
Axial symmetry

$$
\begin{aligned}
ds^2 &= -e^{2\mathrm{U}}\mathrm{N(r)}\,dt^2 + e^{-2\mathrm{U}}dl^2,\\
dl^2 &= e^{2\mathrm{K}}\left[\frac{d\mathrm{r}^2}{\mathrm{N(r)}} + \mathrm{r}^2 d\vartheta^2\right] + e^{2\mathrm{S}}\,\mathrm{r}^2\sin^2\vartheta d\varphi^2,\\
\mathrm{W} &= \mathrm{T}_2\,(F_1\,d\mathrm{r} + F_2\,d\vartheta) - \frac{n}{2}\,(\mathrm{T}_3\,F_3 - \mathrm{T}_1 F_4\,)\,d\varphi\,,\\
B &= -(n/2)\,Y\,d\varphi\,, \quad \Phi^{\mathrm{tr}} = (\phi_1, \phi_2).
\end{aligned}
$$

Here $\mathrm{U}, \mathrm{K}, \mathrm{S}, F_1, F_2, F_3, F_4, Y, \phi_1, \phi_2$ are 10 real functions of r, ϑ that fulfill 10 elliptic equations (constraints are respected).
For non-extremal solutions $\mathrm{N(r)} = 1 - \mathrm{r_H}/\mathrm{r}$ where $\mathrm{r_H}$ labels the solutions. We use the FreeFem++ numerical solver.

Hairy black holes

Same magnetic charge $P = n/(2e)$ as for the RN black hole. Starting at $\mathrm{r_H} = \mathrm{r_H^0}(n)$ when hairy solutions merge RN, we decrease $\mathrm{r_H}$. The massive hair appears and gets longer as the horizon shrinks. When the field at the horizon increases up to $B(\mathrm{r_H}) = m_\mathrm{h}^2$, the hair stops growing and a bubble of symmetric phase appears. This bubble expands as the horizon shrinks further till reaching the minimal value when it becomes degenerate, surface gravity vanishes, but the area remain finite. The black hole then becomes extremal.



Charge contained in the hair

The total magnetic charge P of the black hole splits as

$$P_{\rm h} = \int_{\rm r>r_H} \tilde{\mathcal{J}}^0 \sqrt{-{\rm g}}\, d^3x, \quad P_{\rm H} = P - P_{\rm h},$$

where $P_{\rm h}$ is contained in the hair outside the horizon and $P_{\rm H}$ remains inside. The hair charge $P_{\rm h}$ grows when the horizon shrinks and in the extremal limit one has

$$P_{\rm h} = g'^2 P = 0.22\, P$$

hence 22% of the charge moves to the hair.

ADM mass

is determined from the asymptotic $g_{00} = -1 + 2M/r + \ldots$ or from the formula (same result)

$$M = \frac{k_H A_H}{4\pi} \quad + \quad \frac{\kappa}{8\pi} \int_{r>r_H} \left(-T^0_{\ 0} + T^k_{\ k} \right) \sqrt{-g}\, d^3x,$$

$$\text{surface gravity :} \quad k_H \quad = \quad (1/2)\, N' e^{2U-K}\Big|_{r=r_H}$$

$$\text{horizon area :} \quad A_H \quad = \quad 2\pi r_H^2 \int_0^\pi e^{K+S-2U} \sin\vartheta\, d\vartheta \Big|_{r=r_H}$$

This can be split as

$$M = M_H + M_h$$

where the "horizon mass" M_H is the mass of the RN black hole with the same area A_H and with the charge P_H. The rest is the "hair mass" $M_h = M - M_H$. When the horizon gets smaller, the hair mass M_h and hair charge P_h increase.

Horizon oblatness

The configurations are not spherical, one can define

$$
\begin{aligned}
\text{horizon radius :} \quad r_h &= \sqrt{\mathrm{A_H}/(4\pi)} \\
\text{equatorial radius :} \quad r_{\mathrm{H}}^{\mathrm{eq}} &= \sqrt{\mathrm{g}_{\varphi\varphi}(\mathrm{r_H}, \pi/2)} \\
\text{polar radius :} \quad r_{\mathrm{H}}^{\mathrm{pl}} &= (1/\pi) \int_0^{\pi} \sqrt{g_{\vartheta\vartheta}(\mathrm{r_H}, \vartheta)}\, d\vartheta \\
\text{horizon oblateness :} \quad \delta &= r_{\mathrm{H}}^{\mathrm{eq}} / r_{\mathrm{H}}^{\mathrm{pl}} - 1
\end{aligned}
$$

As the horizon radius decreases, the oblateness δ stars from zero and increases, then reaches a maximum, starts decreasing and approaches zero in the extreme limit. The extremal horizon is perfectly spherical, although the hair is squashed.

Magnetic fields

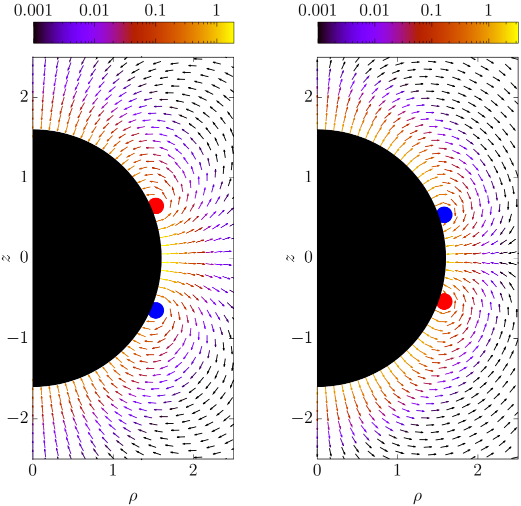


Figure: Magnetic field (left) and the massive magnetic field (right), for the non-extremal solution with $n = 10$, $\mathrm{r_H} = 1.6$ and $\kappa = 10^{-3}$.

Quadrupole moments

Far away from the horizon the theory reduces to electrovacuum,

$$\mathcal{L} = \frac{1}{2\kappa} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}.$$

Writing the metric as

$$\begin{aligned} ds^2 &= -e^{2\mathrm{U}} dt^2 + e^{-2\mathrm{U}} dl^2, \\ dl^2 &= e^{2\mathrm{K}}(d\mathrm{r}^2 + \mathrm{r}^2 d\vartheta^2) + \mathrm{r}^2 e^{2\mathrm{S}} \sin^2\vartheta \, d\varphi^2 , \end{aligned}$$

dualizing the magnetic field, $\sqrt{\frac{\kappa}{2}} F_{ik} = e^{-2\mathrm{U}} \sqrt{h}\, \epsilon_{iks} \partial^s \Psi$, the Ernst potential $\mathcal{E} = e^{2\mathrm{U}} - \Psi^2$. Passing to the Weyl coordinates where $dl^2 = e^{2K(\rho,z)}(d\rho^2 + dz^2) + \rho^2 d\varphi^2$ and considering the asymptotic expansions at the symmetry axis of

$$\xi = \frac{\mathcal{E} - 1}{1 + \mathcal{E}} = \sum_{k \geq 0} \frac{a_k}{z^{k+1}}, \qquad q = -\frac{2\Psi}{1 + \mathcal{E}} = \sum_{k \geq 0} \frac{b_k}{z^{k+1}},$$

gravitational and magnetic quadrupoles are $Q_{\mathrm{G}} = a_2$, $Q_{\mathrm{M}} = b_2$.

Non-extremal hairy solutions

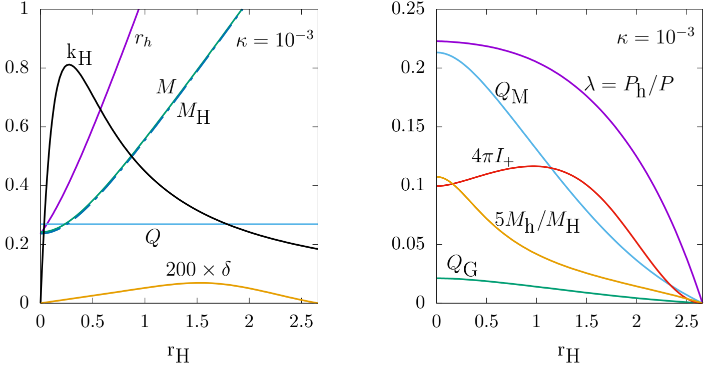


Figure: Parameters of non-extremal solutions with $n = 10$, $\kappa = 10^{-3}$. The M and M_H curves are very close to each other. For $\mathrm{r_H} \to 0$ they become extremal, for $\mathrm{r_H} \to 2.66$ they loose hair and become RN.

Extremal hairy solutions

They have zero surface gravity and are the most hairy. Depending on the value of their charge parameter

$$Q = \sqrt{\frac{\kappa}{2}}\,P = \sqrt{\frac{\kappa}{8}}\,\frac{n}{e} \quad \text{compared to} \quad Q_{\star} \approx \frac{0.72}{\sqrt{\kappa}},$$

there are two phases,

$$\text{phase I}: \quad Q < Q_{\star}, \quad \Phi(r_h) = 0, \quad \text{spherical horizon}$$

$$\text{phase II}: \quad Q > Q_{\star}, \quad \Phi(r_h) \neq 0, \quad \text{non-spherical horizon}$$

In phase I one has $B(r_h) > m_{\rm h}^2$.
In phase II one has $B(r_h) < m_{\rm h}^2$.

Extremal hairy solutions in phase I

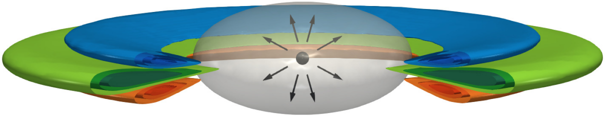


Figure: The extremal solutions contain a small charged black hole inside a bubble of symmetric phase, surrounded by a ring-shaped EW condensate supporting 22 % of the total magnetic charge and two opposite superconducting W-currents. This creates pieces of two magnetic multi-vortices along the positive and negative z-directions. Farther away the condensate disappears and the magnetic field becomes radial.

Electroweak corona

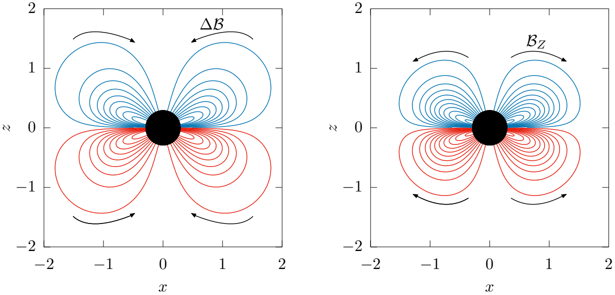


Figure: Field lines of $\mathcal{B} - \mathcal{B}_{\mathrm{RN}}$ (left) and $\mathcal{B}_Z$ (right) for the extremal hairy solution with $n = 10$ and $\kappa = 10^{-3}$ forming two vortices. Each vortex starts in the polar region, extends outwards, and then returns to the horizon in the equatorial region.

Spherically symmetric asymptotics:

$$ds^2 = -N(r)\,dt^2 + \frac{dr^2}{N(r)} + r^2(d\vartheta^2 + \sin^2\vartheta\,d\varphi^2)$$

horizon: the Higgs vanishes $\Rightarrow$ extremal RN-de Sitter $r_{\rm ex} \approx g|Q|$,

$$N(r) = \left(1 - \frac{r_{\rm ex}}{r}\right)^2 \left(1 - \frac{\Lambda}{3}\left[r^2 + 2rr_{\rm ex} + 3r_{\rm ex}^2\right]\right)$$

infinity: the horizon the geometry approaches RN with

$$N(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \mathcal{O}(1/r^3)$$

where

$$Q^2 = \frac{\kappa n^2}{8e^2} \quad \text{and} \quad \boxed{M < |Q|}$$

Normally for RN $M \geq |Q|$ $\Rightarrow$ condensate lowers the mass.

Weakness of gravity

One has $M = M_{\rm H} + M_{\rm h}$ and $Q = Q_{\rm H} + Q_{\rm h}$ with $Q_{\rm h} = 0.22 \times Q$. The hair mass $M_{\rm h} \lll M$ is very small due to the negative Zeeman energy of the condensate interacting with the magnetic field of the black hole:

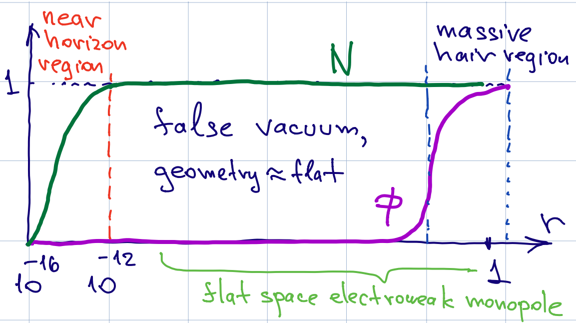
$$m_{\rm w}^2 \to m_{\rm w}^2 - |{\rm B}| \approx 0 \quad \Rightarrow \quad \frac{M_{\rm h}}{|Q_{\rm h}|} \lll 1.$$

The gravity is the weakest force – manifestation of the weak gravity conjecture. The condensate is magnetically repelled by the black hole stronger than attracted gravitationally, but it cannot fly away because it has to follow the Yukawa law. One has

$$M = M_{\rm H} + M_{\rm h} \approx M_{\rm H} \approx g|Q| = 0.88\,|Q| < |Q|$$

Hairy black hole is less energetic than the RN with $M \geq Q$
$\Rightarrow$ they cannot loose the hair and become RN.

Hairy black hole = monopole regularized by gravity



$$ds^2 \approx -N\,dt^2 + \frac{dr^2}{N} + r^2(d\Theta^2 + \sin^2\Theta\, d\varphi^2)$$

near-horizon region is parametrically small as compared to the hair region. The hair lives in flat geometry and does not see the black hole, but the latter renders the mass finite by putting the cutoff at the horizon.

Hairy black holes as magnetic monopoles

- Most popular magnetic monopoles of t'Hooft-Polyakov are not described by the Standard Model.
- Standard Model admits electroweak monopoles, but in flat space their energy diverges because $B \sim n/(2r^2)$. This divergence might be cured by renormalization, but so far nobody has confirmed this.
- Gravity converts electroweak monopoles to hairy black holes and renders their mass finite:

$$M \approx 5.1\,|n|\,M_{\rm Pl}$$

Therefore, if there is no new physics, then these black holes are the only magnetic monopoles which may exist in Nature. They are heavy $\Rightarrow$ should be observed at very high energies

Increasing the black hole charge Q

- The hair length grows till ~ 1 cm, the horizon value of the hypermagnetic field decreases, and when $|Q|$ becomes smaller than $Q_\star$ the Higgs deviates from zero at the horizon.
- The system enters phase II where the horizon is squashed. Near the transition point one has (with $s \approx 10.8$ if $\kappa = 10^{-2}$)

$$\delta \propto (|Q| - Q_\star)^s$$

- The fraction of the hair charge starts to decrease, the black hole looses hair, the geometry approaches extreme RN and finally merges with it for

$$Q_{\rm max} = 2.15\, Q_\star$$

No hairy solutions for $Q > Q_{\rm max}$.

Existence diagram

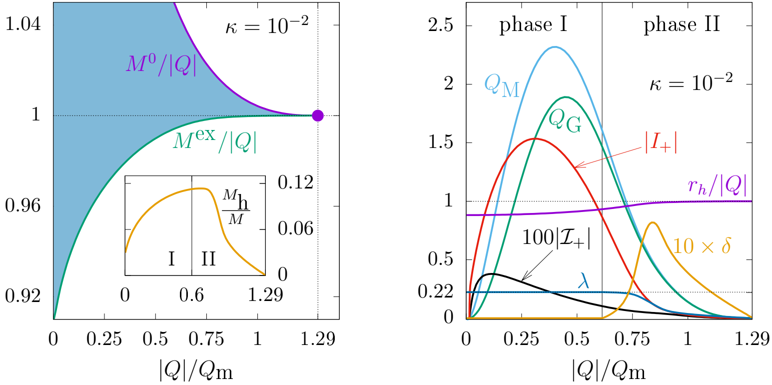


Figure: The parameters of extremal solutions (right) and the existence diagram for hairy solutions (left) for $\kappa = 10^{-2}$; $Q_m = 1/(g\sqrt{\Lambda})$.

Maximally hairy extremal black hole

The black hole is maximally hairy around the phase transition point for $|Q| \approx Q_\star$, when the fraction of the hair mass M_h/M is maximal. Then

$$|n| \approx 1.5 \times 10^{32}, \qquad r_h \approx 1.37 \text{ cm}, \qquad r_c \approx 2.2 \text{ cm},$$

the black hole mass has a planetary value,

$$M \approx 2 \times 10^{25} \text{ kg}$$

of which $\approx 11\%$ is contained in the hair condensate.

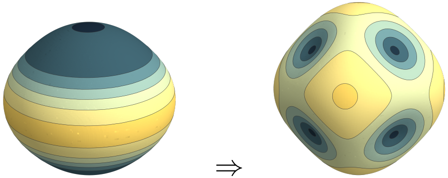
The magnetic charge with $|n| \approx 1.5 \times 10^{32}$, if put at the Sun center, would create at the Sun surface a field of $\approx 10^3$ Gauss, which is several times smaller than in sunspots.

Stability

According to Maldacena, the corona enhances the Hawking evaporation rate, hence non-extremal black holes should quickly relax to the extremal state when their temperature is zero and

$$M < |Q|$$

Therefore, they cannot decay into RN black holes. However, axially symmetric black holes can further reduce their mass by splitting their hair into a hedgehog of vortices – “spreading the corona”. Then the condensate energy achieves an absolute minimum and the hairy black holes seem to become absolutely stable. The corresponding solutions have not yet been obtained.



Phenomenology

Since they are described by well-tested theories, the hairy EW black holes are expected to be physically relevant. They could probably originate from primordial black holes. If the fluctuating magnetic field in the ambiant EW plasma becomes at some moment mostly orthogonal to the black hole horizon, or a piece of a magnetic vortex gets attached to the horizon, this creates a flux through the horizon = charge. This flux should be compensated by the opposite flux created on other black hole(s). The oppositely charged black holes will not necessarily annihilate, being pushed apart by the cosmic expansion, or maybe they form bound systems stablized by the scalar repulsion.

Such black holes should catalize the proton decay. They can be detected when captured by a neutron star, causing a sudden change of the star's rotation period. Estimates based on proton decay and Parker bound show that their contribution in Dark Matter is small, unless they form neutral bound systems.

Conclusions

- We constructed for the first time hairy black holes described by well-tested theories, GR and SM. This suggests that they may really exist in Nature. Perhaps they could have been created by fluctuations in primordial electroweak plasma.
- The can be as large as ≈ 1 cm with approximately terrestrial mass $M \sim 10^{25}$ kg of which $\approx 10\%$ is stored in the electroweak condensate hair.
- They have $M < |Q|$ hence cannot get rid of the hair and evolve into RN. When they spread their corona, they lower the energy to an absolute minimum and seem to become absolutely stable. This minimum is presumably highly degenerate – the hair carries entropy.
- If there is no new physics beyond the Standard Model, then they should be the only magnetic monopoles which may exist.