A PORTRAIT OF LICHNEROWICZ
Mathematician, Physicist, Mentor

Introduction

I shall speak of the life of Lichnerowicz in a brief biography and then I shall treat some aspects of his varied and extremely large bibliography, emphasizing spinors and deformation quantization. I shall recall some personal memories of my work under his direction.

1. The career of André Lichnerowicz

André Lichnerowicz was born on 21 January, 1915. Whence came his polish name? His grandfather had to flee Poland for resisting the Prussians in 1860, he took refuge in France and settled there. He married an heir to a distinguished French family, who originally manufactured paper. His other two grandparents were from north-western France

Both his parents were teachers, both professeurs agrégés. His father taught classics (Latin and Greek), while his mother was among the early agrégées in mathematics.

He was admitted to the École Normale Supérieure at age 18 and succeeded in the "agrégation" at 21. He was a student of Élie Cartan (1869-1951) and Georges Darmois (1888-1960), a distinguished differential geometer whose work on relativity theory dates frm 1923-1933 and who later made important contributions to probability theory. After that, Lichnerowicz had a swift career. He entered the CNRS in 1937, submitted his doctorate in 1939, remained for two more years at the CNRS as a chargé de recherches. Soon he obtained a position in Strasbourg, first as maître de conférences, then as professor of Rational Mechanics. He remained in Strasbourg from 1941 to 1949, with an episode in Clermont-Ferrand to which city, in un-occupied France, the University of Strasbourg was moved during the war. Laurent Schwartz, in his autobiography, speaking of that time, tells the following story: "Sometimes Lichnerowicz would enter a police station on some pretext or other and take advantage of a momentary absence of the policeman at the desk to hastily snatch his stamp and stamp one of our cards. The situation of the Jews demanded prodigious inventiveness at all times."

After Strasbourg, Lichnerowicz was professor in Paris for three years and then he was elected in 1952 to a chair at the Collège de France, at the early age of 37. He held the chair of Mathematical Physics until he reached the retirement age in 1986.

In 1962, he became an associate member of the Academia Nationale dei Lincei.

In 1963, at age 48, again very young by the standards of the time, he was elected a member of the Académie des Sciences. His students collected money to offer him his academician sword, a tradition for newly elected members.

In 1965, for his fiftieth birthday, his students offered him ... a pipe. He can be seen in nearly all photographs with his pipe!

Lichnerowicz was awarded several doctorates honoris causa and the high-

est honor (commandeur) in the Légion d'Honneur among very many awards. He was invited to lecture in innumerable countries all over the world.

In 1976, a volume "In Honour of André Lichnerowicz on His 60th Birthday" was edited by Moshé Flato and Michel Cahen. This book contains important papers on both differential geometry and mathematical physics, together with a detailed preface by the editors. It contains also a contribution by Robert Couty and André Revuz on "thirty years of activity in the renovation of mathematical education". The role of Lichnerowicz in this area was very important. In Caen in 1956 and in Amiens ten years later he was already participating in conferences on the reform of education. He chaired what everyone called the "Lichnerowicz commission" (Comité pour l'Enseignement des Mathématiques) from 1966 to 1973 as well as the committee of the IREM's or Instituts de Recherche sur l'Enseignement Mathématique.

He kept publishing articles on mathematics, physics, philosophy and education. As counted in Zentralblatt, the number of his publications - articles and books, published alone or with collaborators - is 360.

He died suddenly, at age 83, on 1 December, 1998.

As Professor at the Collège de France, Lichnerowicz gave two lessons every week, on subjects that had to be different from year to year, on differential geometry as well as in mathematical physics. He delivered these lectures in a deliberate, very clear enunciation. He would reproduce his long calculations on the blackboard, in his elegant, vertically elongated, hand-writing. In his courses, he dealt with deep subjects, always maintaining a very clear expositi on. His topics ranged from relativistic magnetohydrodynamics to contact homogeneous spaces, as well as deformations of associative and Lie algebras, Poisson geometry and Dirac brackets, foliations, Kähler+ manifolds, and many other subjects. On the website of the Collège de France, you can read his very informative summaries of his courses.

2. Lichnerowicz's work in General Relativity

The work of Lichnerowicz in general relativity - the relativistic theory of gravitation, the Einstein equations and, more generally, relativistic mechanics - extends over a long period, beginning with his first publication in 1937. In 1958, he introduced the notion of propagators. I shall not dwell on the subject because you can read a detailed account of his results in this area in the article witten by Yvonne Choquet-Bruhat in the volume "Physique quantique et géométrie" which contains the proceedings, published by Hermann, of the International Colloquium "Géométrie et Physique" in honor of André Lichnerowicz, held in 1986.

3. Lichnerowicz's work in Riemannian Geometry

I shall not speak either of the work of Lichnerowicz in Riemannian geometry because a 14-page survey of a large part of it (with references to more work of Lichnerowicz) is in Marcel Berger's contribution to the abovementioned book which was published when Lichnerowicz turned 70.

This same volume contains the contribution of Charles-Michel Marle on the works of Lichnerowicz on symplectic geometry, as well as Poisson and Jacobi manifolds, canonical manifolds and contact manifolds, with a summary of his work on deformations, of which more below.

4. Lichnerowicz's work on spinors

In 1961, Lichnerowicz published a first Note on the subject of spinors in the Comptes rendus, "Anticommutateur du champ spinoriel en relativité générale", followed in close succession by two other Notes. Then in 1962 came his article, "Laplacien sur une variété riemannienne et spineurs" and the followig year, he published "Spineurs harmoniques" which proved to be fundamental. These contributions were followed, in 1964, by "Champs spinoriels et propagateurs en relativité générale", where he cites a 1941 paper by Rarita and Schwinger on particles with half-integer spin.

Jean-Pierre Bourguignon gave both an account of the work of Lichnerowicz on spinors and of his own relationship with him. He also described his subsequent work on spinors with Paul Gauduchon. So I shall not dwell on the subject. I shall rather give personal recollections of my interaction with Lichnerowicz.

4bis. My recollections

My first visit, when I decided to solicit advice and to ask him to become my adviser for a thesis, was in late 1963 or in 1964 when I was a fourth-year student at the École Normale Supérieure de Jeunes Filles - also called Sèvres, from its location before the war. (There were still separate schools for boys and girls at the time and it took until 1975 for the two schools to be united. At Sèvres, the agrégation took place in the third year and a few of us obtained a fourth year of study,but we were no longer boarders at the school on Boulevard Jourdan.)

At that time, Lichnerowicz directed me to read Bogolioubov and Chirkov, Introduction à la théorie quantique des champs, a voluminous book more than 600 pages long, a treatise on quantum mechanics which had recently appeared in 1960. (It was a French translation of the English translation of 1959, which had been itself a translation of the earlier Russian edition.) I was impressed but did not feel interested by the little I made of this thick book. So I sought another appointment with him.

The approximately monthly meetings took place either in his office, a small room on the upper floor of the Collège de France, under the roof, or in the office of his apartment, 6 avenue Paul Appell, on the southern edge of Paris, which overlooked a vast expanse of sports grounds.

When I reported on the little I had read and understood in Bogoliubov and Chirkov, he suggested another topic: spinors, still at the border between physics and mathematics. I started working on spinors immediately and published a first Note aux Comptes rendus de l'Académie des Sciences in December 1966, "Dérivées de Lie des spineurs", immediately followed by a second one. These were followed by a third Note aux Comptes rendus the following year. Thus he directed my thesis, which was the thèse d'état, which usually took from 4 to 6 years, or more, to complete.

In 1969, I brought out a roneotyped copy of my work, printed with special permission in the library of the IHP, then situated on the fourth and last floor, which was headed by Belgodère, renowned for his knowledge and bad temper, and aided to maintain strict discipline in the small public section of the library by Mademoiselle Lardeux.

At that time, I took part in a study group that met in the basement of the Institut Henri Poincaré. It was a very small group of students, men and women, whose work was supervised by Lichnerowicz. His students and his friends called him Lichné.

My defense was eventually scheduled for May 1970, with Dixmier, Zisman and Mlle Libermann in the Jury, which was presided by Lichnerowicz. (Paulette Libermann had arrived as a professor in Paris in 1966. We first met at the thé des mathématiciens, and she soon invited me to speak in her seminar.) My thesis was sent to Annali di Matematica in April 1971 and published only a year later, another delay which was due to the fact that I had accepted an invitation to travel to Madagascar in early 1971 to teach a course, I had taken the proofs along with me to the Île Sainte Marie and had read them by candle light in the cabin that served as a hotel room on the incredibly beautiful beach where palm-trees grew.

5. Lichnerowicz, Flato and Quantum Mechanics

From the theory of spinors, Lichnerowicz moved to quantum mechanics. Rather than going into this theory with well-known developments, I shall mention the deep and unexpected friendship that linked Lichnerowicz with Moshé Flato and I shall enumerate their joint publications.

Flato was an extraordinary person, portrayed in Conférence Moshé Flato 1999 (volume 1, Kluwer 2000) by Daniel Sternheimer. Flato came to France from Israel in 1963. His personality was completely opposed to that of Lichnerowicz, who had a steady approach to mathematical physics and, I would say, an aristocratic demeanor. They met early as Flato settled in France. Lichnerowicz was president of Flato's thesis defense in 1967. Flato gave a seminar at the Collège de France, invited by Lichnerowicz, as early as 1967. Their collaboration began a few years later. They soon become close friends, and called one another André and Moshé. Together, Lichnerowicz and Flato wrote 11 articles, and they remained close friends until the untimely death of Flato on 27 November 1998, followed less than a month later by that of Lichnerowicz, on 11 December.

Below is a short quotation from the talk of Jérôme Lichnerowics, the son of André, during the Conférence Moshé Flato (in a free translation from the French): "I shall speak of the friendship that has bound Moshé Flato and my father, André Lichnerowicz, for more that thirty-five years. [...] There was no way of disassociating them in science as in life: they remained so until the end. We miss them in their similarities, their differences, their complementarity, their absence."

Since 1964, Lichnerowicz had been working and publishing, among many other subjects, on complex manifolds, symplectic and contact manifolds , Kählerian geometry, Poisson manifolds and their deformations.

The first joint papers of Lichnerowicz with Flato and Sternheimer are dated 1974, 1975 and 1976, "and led to spectacular developments with the deformation quantization program", to cite Simone Gutt's comprehensive article published in Conférence Moshé Flato. Their collaboration culminates in Bayen, Flato, Fronsdal, Lichnerowicz, and Sternheimer, Deformation quantization, I and II, published in Annals of Physics in 1978. (A short announcement by the same authors had been published in Letters in Mathematical Physics in 1977.) Part I was on the deformations of symplectic structures and Part II dealt with physical applications. There they discovered and described star-products, that is deformation quantization.

In collaboration with Flato, Lichnerowicz wrote a further article in 1980, in French, in the Comptes rendus, with a long title, "Cohomologie des représentations définies par la dérivation de Lie et à valeurs dans les formes, deconflichne.tex l'algèbre de Lie des champs de vecteurs d'une variété différentiable. Premiers espaces de cohomologie. Applications" (Cohomology of the representations of the Lie algebra of a differentiable manifold defined by form-valued Lie derivation. First cohomology spaces. Applications) Then came two articles by Lichnerowicz alone, one in 1982, in French in the Annales de l'Institut Fourier entitled "Déformations d'algèbres associés à une variété symplectique (les $*_\nu$-produits)" ("Deformations of algebras associated to a symplectic manifold (the $*_\nu$-products)"), and, in 1983, an 80-page article in English, in a book on "Quantum Theory, Groups, Fields and Particles" edited by A. O. Barut, "Quantum Mechanics and Deformations of Geometrical Dynamics".

Again in 1984, Lichnerowicz published two articles with Henri Basart, Flato and Sternheimer, this time on statistical mechanics, "Déformations et Mécanique statistique", in the Comptes rendus, and "Deformation Theory applied to Quantization and Statistical Mechanics", in Letters in Mathematical Physics. Then four more articles with Basart, all in French, all on statistical mechanics, another subject of the research of Lichnerowicz.

6. His students (in no particular order)

Yvonne Choquet-Bruhat, who herself authored 200 publications and books and became a member of the Académie des Sciences in 1979.

Charles-Michel Marle

Marcel Berger

Thierry Aubin

Thibault Dammour

Paul Gauduchon

Jean-Marie Souriau

Pierre Molino

Richard Kerner who came to Paris from Poland in 1965 and completed his thesis in 1975

Michel Cahen

Yvan Kerbrat who was professor in Lyon

Hélène Kerbrat-Lunc who published or co-published 19 papers from 1964 to 1992

Daniel Lehmann

Josiane Lehmann-Lejeune

Claude Latrémolière

Éric Lehman

Claudette Butin who died prematurely in 1972 before completing her thesis.

A large number of women, unusual for the time, were working with him in the 1960's. Did they complete their dissertations? To judge from The Mathematics Genealogy project, no.

7. The creation of the Lichnerowicz prize

The Licherowicz prize was created in 2008, during the Poisson Geometry conference, in Lausanne. It marked the 10th anniversary of Lichnerowicz's death.

If you look up the "Poisson Geometry" website, you will see both a description of the prize in English, and a list of all the awardees of the prize, beginning with Henrique Bursztyn and Marius Crainic, the year it was first awarded.

To end this lecture marking the 110th anniversay of the birth of André Lichnerowicz, I shall tell the story behind the awarding of the first Lichnerowicz prize. The name of the prize had been chosen and the laureates had been elected by a due process of voting. Then came the day of awarding the prize. We realized that it did not have a description! That is when Tudor Ratiu and I drafted it, at the last minute, with the help of ... my non-mathematician, but English-speaking, husband! Our description has been in use ever since.

Conclusion

Lichnerowicz wrote in 1982, more than 40 years ago:

"During the last fifteen years, two distinctly different approaches, presenting certain interactions, have lent a geometric form to quantum mechanics analogous to that of classical mechanics. I mean on the one hand the geometric quantification of Kostant and Souriau, and, on the other, the theory of star-products due to Moshé Flato, Jacques Vey and myself. The first approach utilizes, as a base space, a symplectic homogeneous space G/H, a covering of an orbit of G, according to a theorem of Souriau, corresponding to its coadjoint representation, à la Kirillov-Kostant-Souriau; it uses real and complex polarizations. The second approach considers quantum mechanics as a nontrivial deformation of classical mechanics in the sense of Gerstenhaber, the deformation parameter being related to Planck's constant. These two methods seem to me to be interesting and promising, but they have, perhaps unjustly, attracted more interest among mathematicians than among physicists. Actually, they pose interesting mathematical and, in particular cohomological problems."

A fine, visionary program, which he himself had already greatly advanced.